

Pythagoras

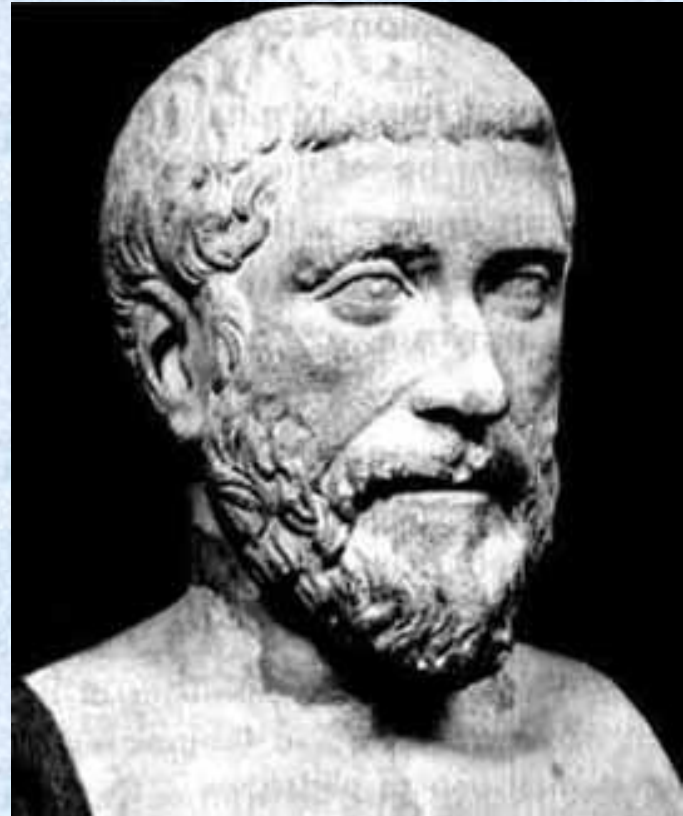
580 - 496 B.C.E.



Pythagoras of Samos

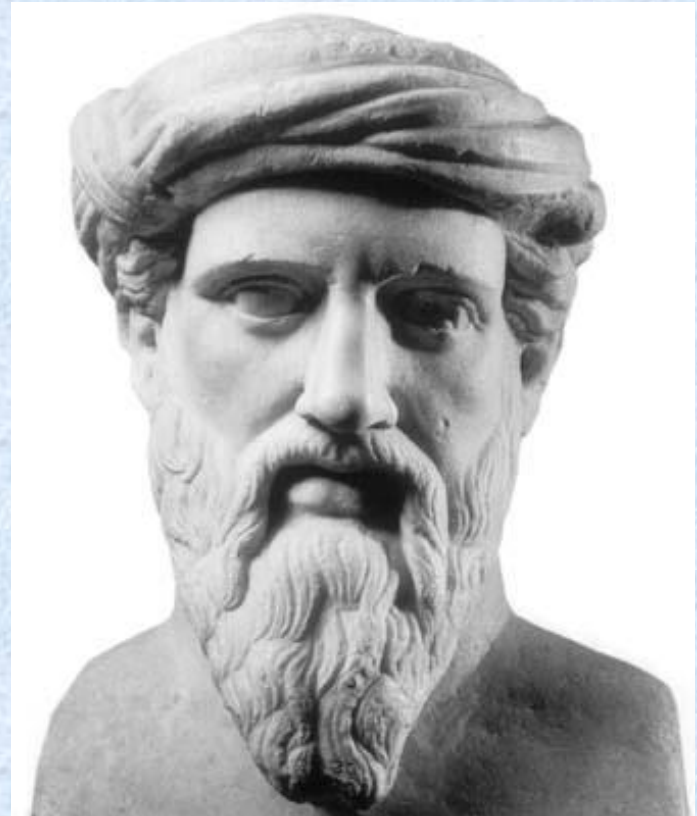
- Little is known about the life of Pythagoras.
- He was born about 569 BC on the Aegean island of Samos.
- Died about 475 BC.
- Studied in Egypt and Babylonia.

- 580 - 496 B.C.E.



Pythagoras of Samos

- Pythagoras founded a philosophical and religious school in Croton (now Crotona, on the east of the heel of southern Italy) that had many followers.



Pythagorean Brotherhood

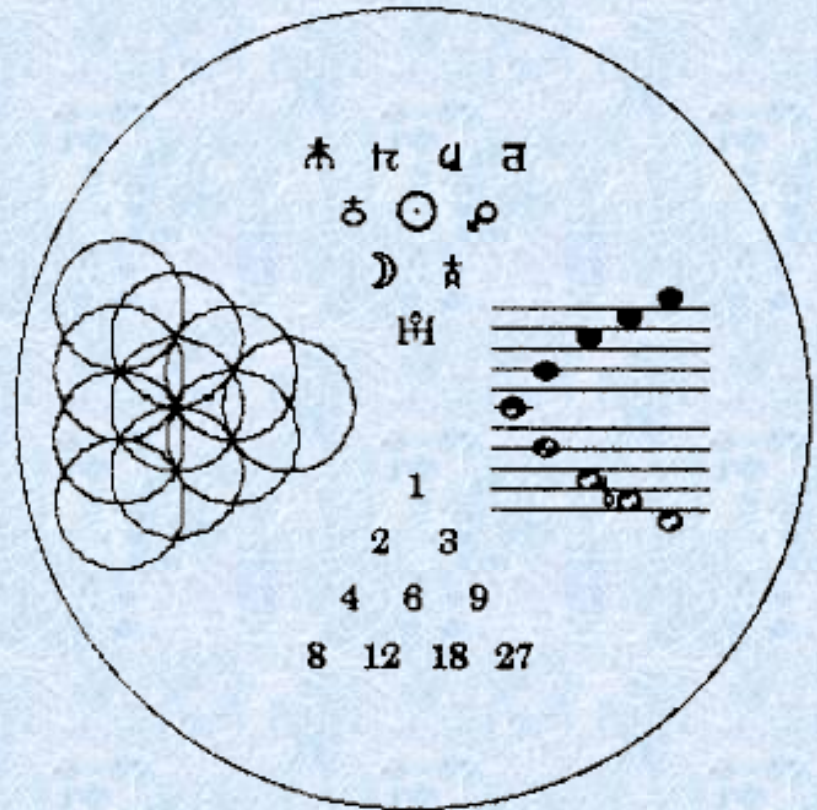
- Pythagoras was the head of the society with an inner circle of followers known as *mathematikoi*.
- The *mathematikoi* lived permanently with the Society, had no personal possessions and were vegetarians.
- They were taught by Pythagoras himself and obeyed strict rules.

Pythagoreans

- Both men and women were permitted to become members of the Society, in fact several later women Pythagoreans became famous philosophers.
- The outer circle of the Society were known as the *akousmatics* (listeners) and they lived in their own houses, only coming to the Society during the day.
- The members were bound not to disclose anything taught or discovered to outsiders.

The Quadrivium

- They studied the “Quadrivium”
 - Arithmetica (Number Theory)
 - Harmonia (Music)
 - Geometria (Geometry)
 - Astrologia (Astronomy)

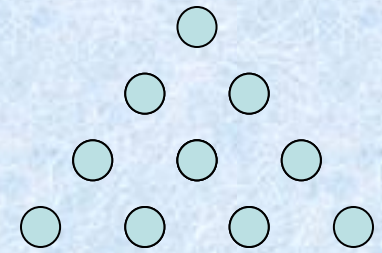


The Seven Liberal Arts

- Later added the “Trivium”,
 - Logic
 - Grammar
 - Rhetoric
- Forming the “Seven Liberal Arts”

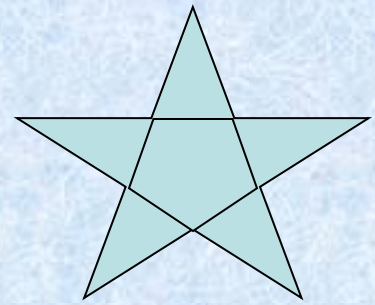


Pythagoreans



- The symbol they swore their oath on was called the “tetractys”.
- It represented the four basic elements of antiquity: fire, air, water, and earth.
- It was represented geometrically by an equilateral triangle made up of ten dots and arithmetically by the sum
– $1 + 2 + 3 + 4 = 10$

Pythagoreans



- The five pointed star, or pentagram, was used as a sign so Pythagoreans could recognize one another.
- Believed the soul could leave the body, i.e., transmigration of the soul.
- “*Knowledge is the greatest purification*”
- Mathematics was an essential part of life and religion.

Pythagoras' Philosophy

- Theorized that everything, physical and spiritual, had been assigned its allotted number and form.
- “*Everything is number.*”
- According to Aristotle, “*The Pythagoreans devoted themselves to mathematics, they were the first to advance this study and having been brought up in it they thought its principles were the principles of all things.*”

- Pythagoras is considered the Father of Numerology, however he used much of information from written texts from the Chaldean Empire as he setup his own system, which we now know as modern day Numerology

Pythagorean Numerology

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

The practice is to total the numerical value of a name or word. For example, the name "John Smith".

JOHN = 1 + 6 + 8 + 5 = 20, SMITH = 1 + 4 + 9 + 2 + 8 = 24,

20 + 24 = 44, 44 = 4 + 4 = 8

Chart of Characteristics of Numbers

1 unity, creation, independence

2 duality, emergence

3 power, generative force

4 solidity, dullness

5 sensuality, pleasure

6 perfection, harmony, balance

7 mysticism, psychic, magic

8 material, success, justice

9 spiritual, mental achievement

Numerology

- A mystical belief that was common in many ancient societies.
- Various numbers represented things like love, gender, and hate.
- Even numbers were female while odd numbers were male.
- The number 1 was the omnipotent One and the generator of all numbers.
- The number 2 was the first female number and represented diversity.

Numerology

- $3 = 1 + 2$ was the first male number composed of unity and diversity.
- $4 = 2 + 2$ was the number for justice since it is so well balanced.
- $5 = 2 + 3$ was the number of marriage.
- Earth, air, water and fire, were composed of hexahedrons, octahedrons, icosahedrons, and pyramids – geometric solids differing in the *number of faces*.

- Music

- He discovered that notes sounded by a vibrating string depended on the string's length.
- Harmonious sounds were produced by plucking two equally taut strings whose lengths were in proportion to one another.
- Proportions related to the tetractys!

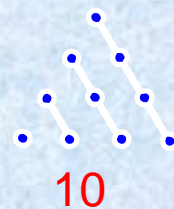


Harmonious Musical Intervals

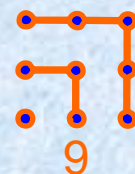
- If one string was twice as long as the other, i.e., their lengths were in ratio 1:2, then an **octave** sounded.
- If their lengths were in ratio 2:3, then a **fifth** sounded.
- If their lengths were in ratio 3:4, then an **fourth** sounded.

- Greeks tried to transfer geometric ideas to number theory
- One of such attempts led to the appearance of polygonal numbers

triangular



square



pentagonal



Some say Luke's two chapters in The Bible has hints that he studied and was aware of
Pythagoreans

Pythagoras did not eat fish or meat and he would not eat beans. The vegetarian diet was actually was called the Pythagorean diet until the 18th century.

Pythagoras also believed in reincarnation.

Mathematics

- The Pythagoreans wrote many geometric proofs, but it is difficult to ascertain who proved what, as the group wanted to keep their findings secret. They did not write anything down until the very end. Unfortunately, this vow of secrecy prevented an important mathematical idea from being made public.
- Legend has it that upon completion of his famous theorem, Pythagoras sacrificed 100 oxen. They were against the killing of animals.
- He felt that “practical” (or applied) **mathematics** there can exist no **irrational** numbers.

DRAMA

- The Pythagorean society considered whole numbers to be their rulers and that all quantities could be explained by whole numbers and their ratios.
- Pythagorean Hippasus who discovered that the diagonal of a square whose side was one unit could not be expressed as a whole number or a ratio.
- Some say that he was drowned, most believe that he was expelled from the Pythagorean Society and many believe he was killed because of his discovery.

Why do we call it the Pythagorean Theorem?

- Pythagoreans were around 500 BCE.



Pythagoreans celebrate sunrise by Fyodor Bronnikov

Babylonian tablet named Plimpton 322



- Ancient clay tablets from Babylonia indicate that the Babylonians in the second millennium B.C., 1000 years before Pythagoras.
- It had rules for generating Pythagorean triples, understood the relationship between the sides of a right triangle, and, in solving for the hypotenuse of an isosceles right triangle
- It came up with an approximation of the square root of 2 that was accurate to five decimal places.

Translation from the Rhind Papyrus - dated from between 1900 and 1600 BCE

4 is the length and 5 the diagonal. What is the breadth ?

Its size is not known.

4 times 4 is 16.

5 times 5 is 25.

You take 16 from 25 and there remains 9.

What times what shall I take in order to get 9 ?

3 times 3 is 9.

3 is the breadth.

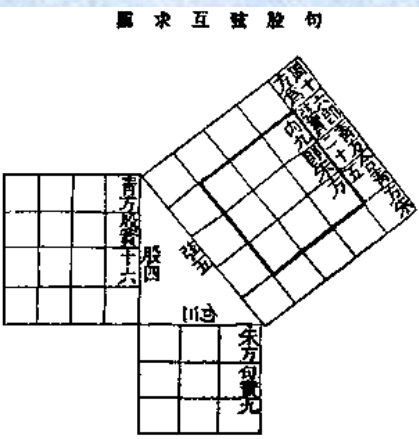
**Its not a
proof!**

LOOKS LIKE PYTHAGOREAN THEOREM!!!!!!!!!!

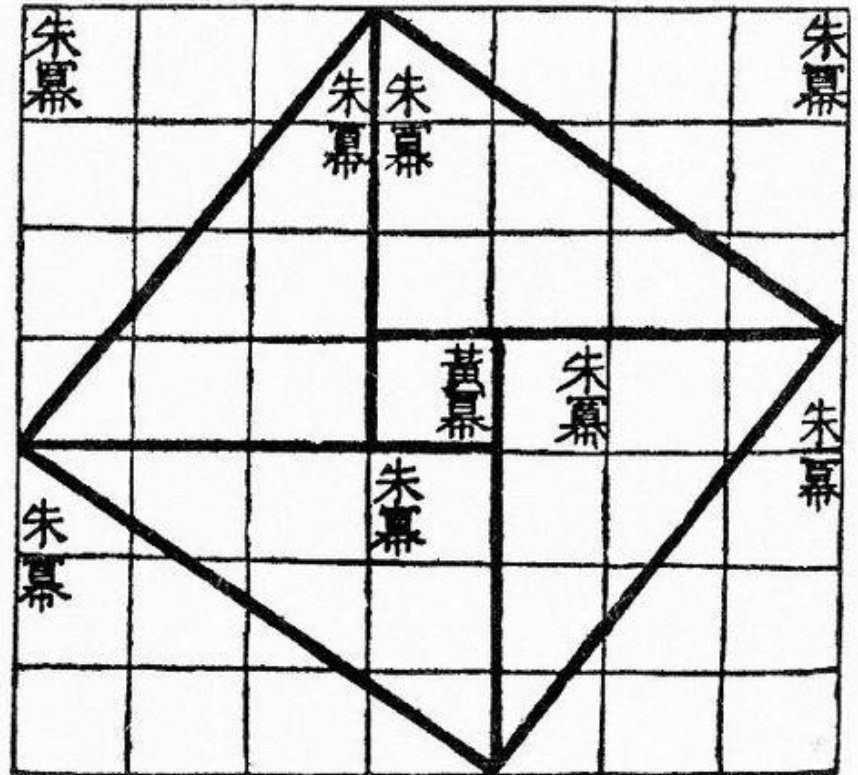
Egyptians most likely used it to build the Pyramids



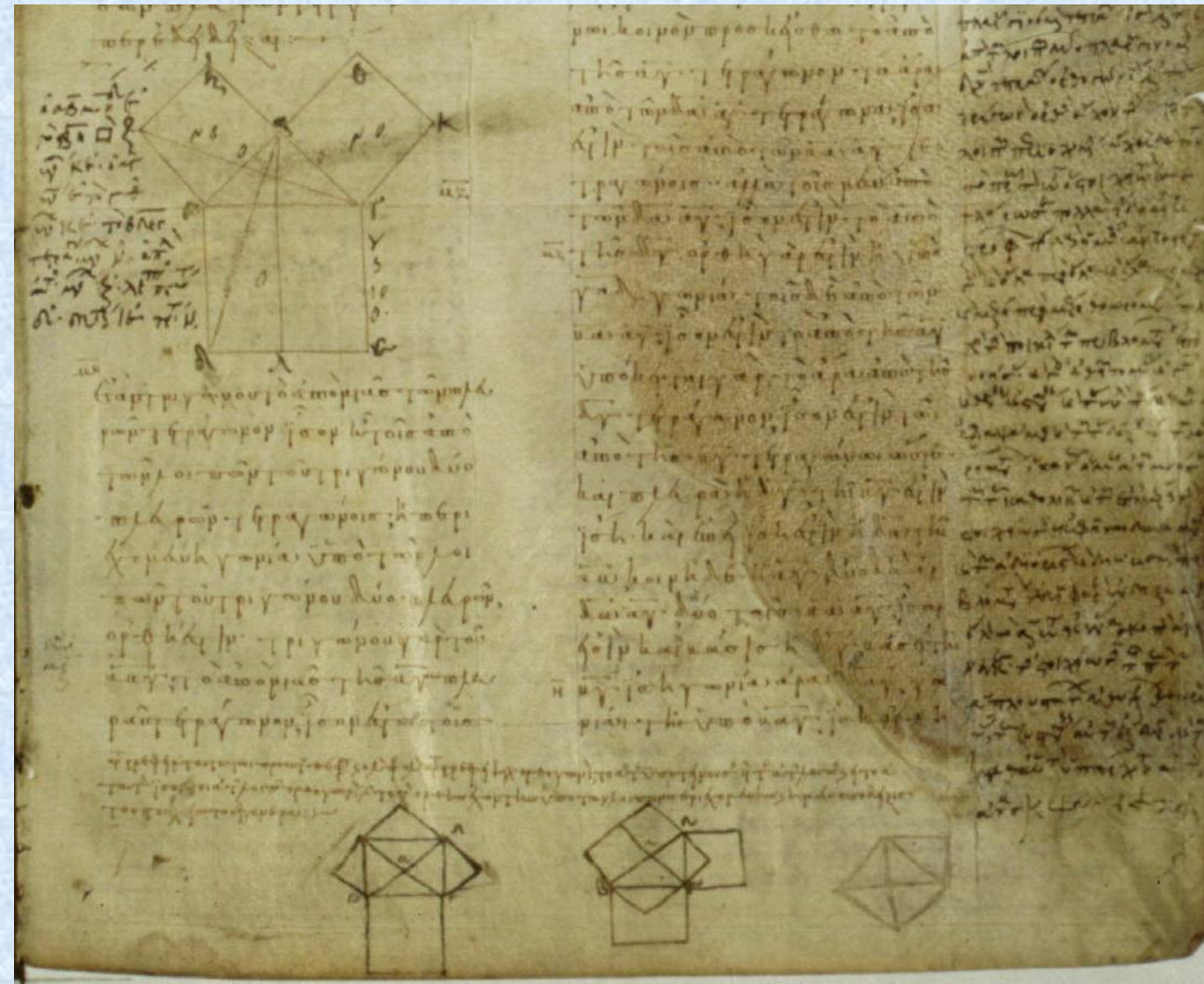
A Chinese astronomical and mathematical treatise called *Chou Pei Suan Ching* (*The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*, ca. 500-200 B.C.), possibly predating Pythagoras, gives a statement of and geometrical demonstration of the Pythagorean theorem



句股冪合以成弦冪

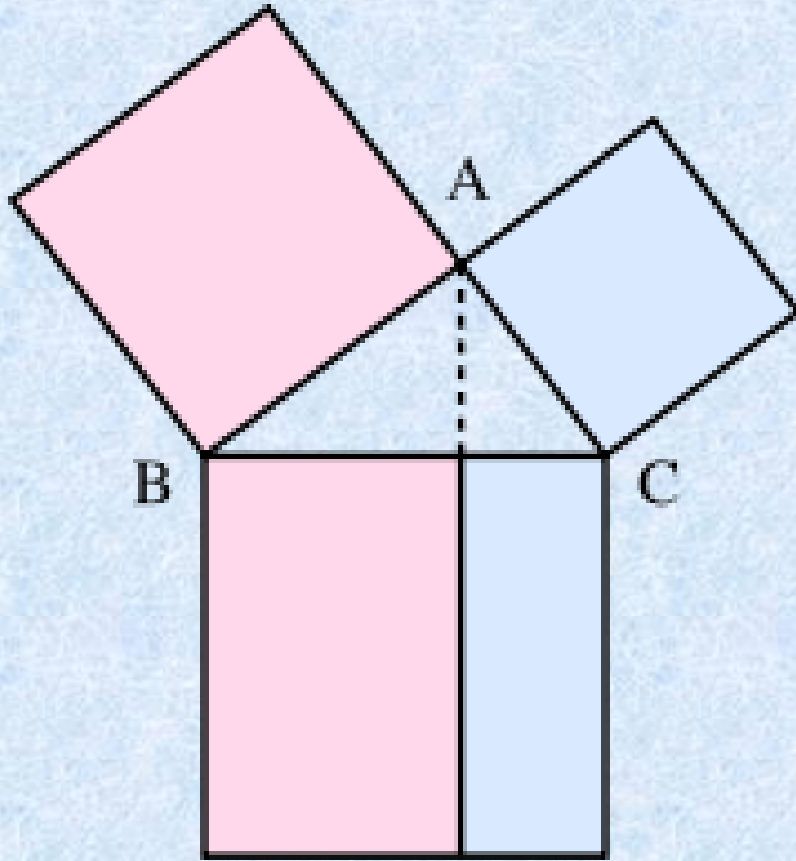


Pythagorean Theorem did not exist until 200 years after his death. Euclid in his book element. This is the copy of Euclid's Elements that is stored at the Vatican.



Euclid attributed Pythagoras with its first geometrical demonstration.

Euclid's Proof



- The idea is to prove that the little square (in blue) has the same area as the little rectangle (also in blue) and etc.
- He does so using basic facts about triangles, parallelograms, and angles.

It wasn't Pythagoras?

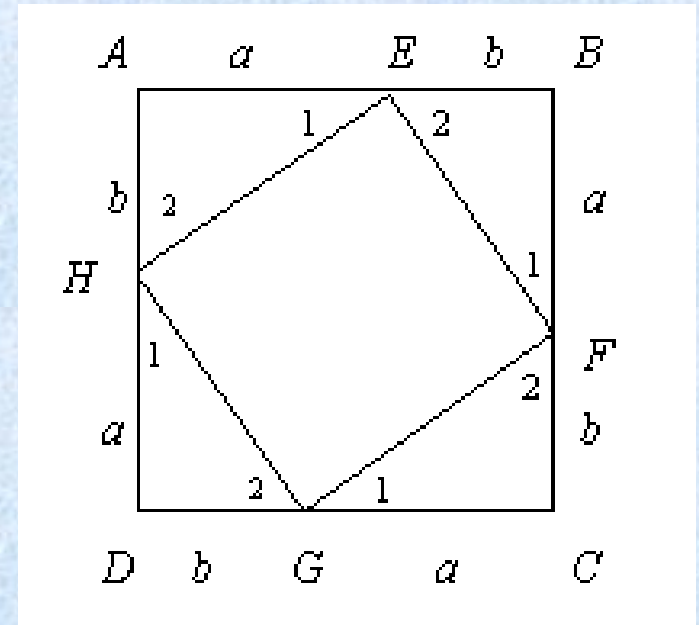
- A common discovery
 - Happened during prehistoric times
- Theorem came “naturally”
 - Independently discovered by multiple cultures
 - Supported by Paulus Gerdes, cultural historian of mathematics
 - Carefully considered patterns and decorations used by African artisans, and found that the theorem can be found in a fairly natural way

Proofs of Pythagorean Theorem

- Whole books devoted to ways of proving the Pythagorean Theorem
- Many proofs found by amateur mathematicians
 - U.S. President James Garfield
 - He once said his mind was “unusually clear and vigorous” when studying mathematics

“Square in a Square”

- Earliest proof, based on Chinese source
- Arrange four identical triangles around a square whose side is their hypotenuse
- Since all four triangles are identical, the inner quadrilateral is a square



“Square in a Square”

$$(a + b)^2 = a^2 + 2ab + b^2$$

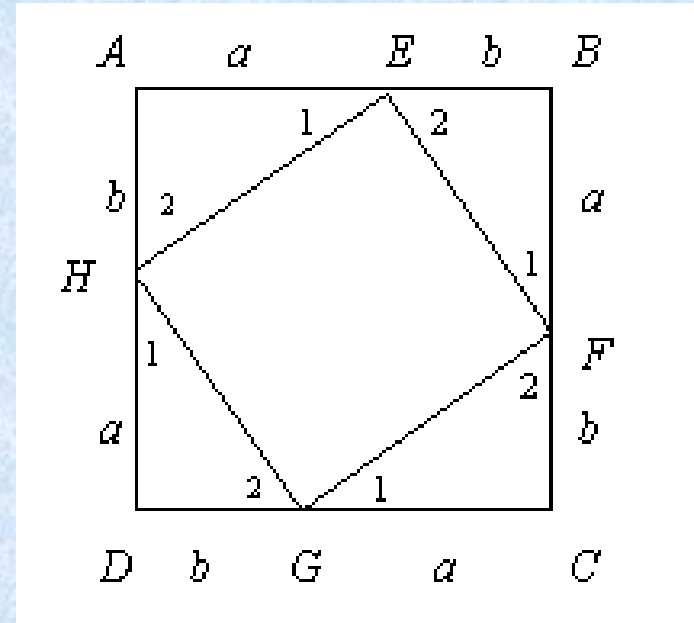
$$(a + b)^2 = c^2 + 4\left(\frac{1}{2}ab\right)$$

Equating these two equations gives

$$a^2 + 2ab + b^2 = c^2 + 4\left(\frac{1}{2}ab\right)$$

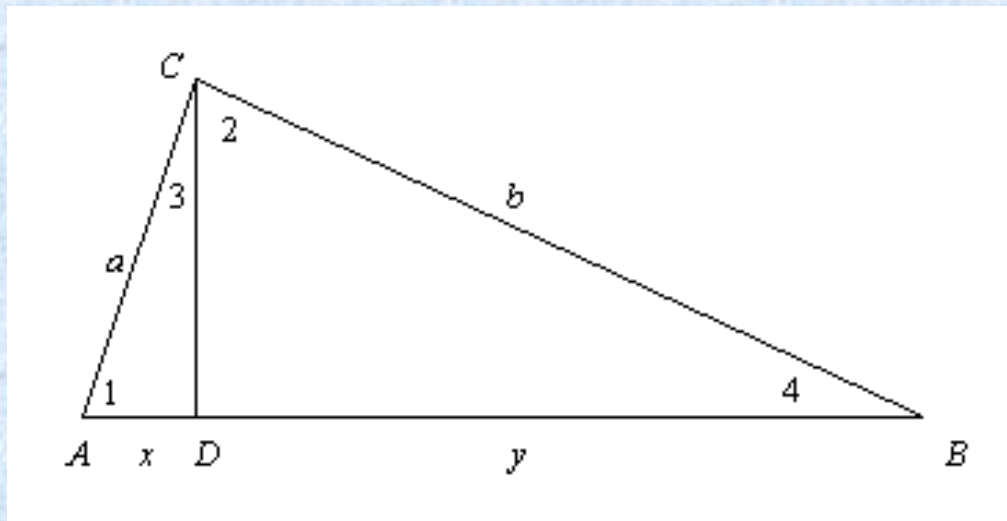
Subtracting the $2ab$ from both sides gives

$$a^2 + b^2 = c^2$$



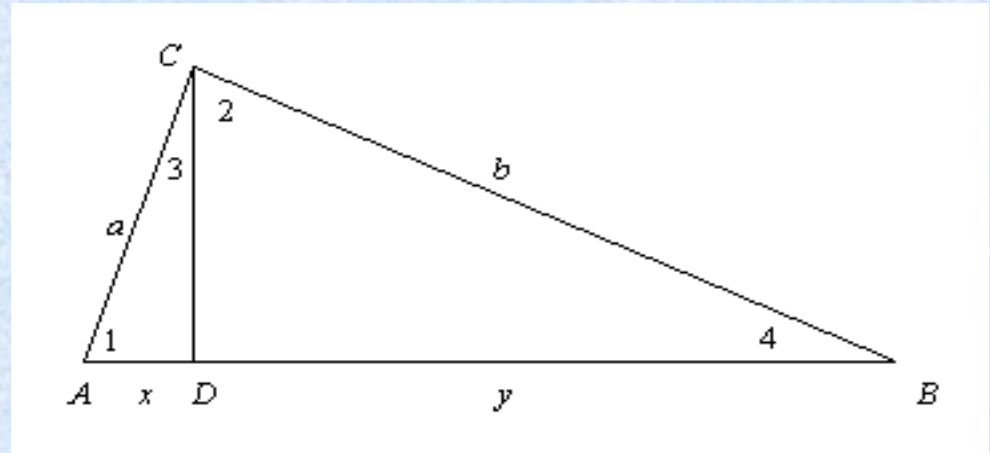
Proof #2

- In right triangle $\triangle ABC$, the altitude CD is *perpendicular* to (makes a 90° angle with) hypotenuse AB .
- AD and DB have lengths x and y which add up to c , the length of the hypotenuse, i.e., $c = x + y$.



Proof #2 (continued)

- $\angle 1 = \angle 2$
- $\angle 3 = \angle 4$
- All three right triangles are similar so certain ratios are equal.
- By comparing triangles $\triangle ACD$ and $\triangle ABC$, we get $a/x = c/a$.
- Comparing triangles $\triangle BCD$ and $\triangle ABC$, gives $b/y = c/b$.



Crossmultiplying gives

$$a^2 = cx$$

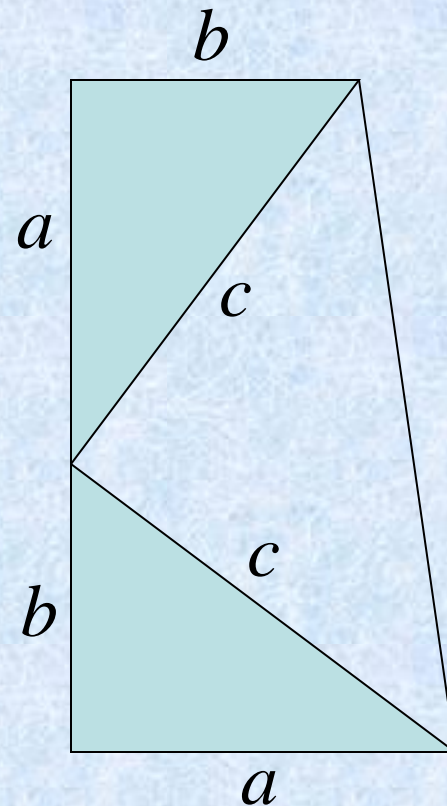
$$b^2 = cy$$

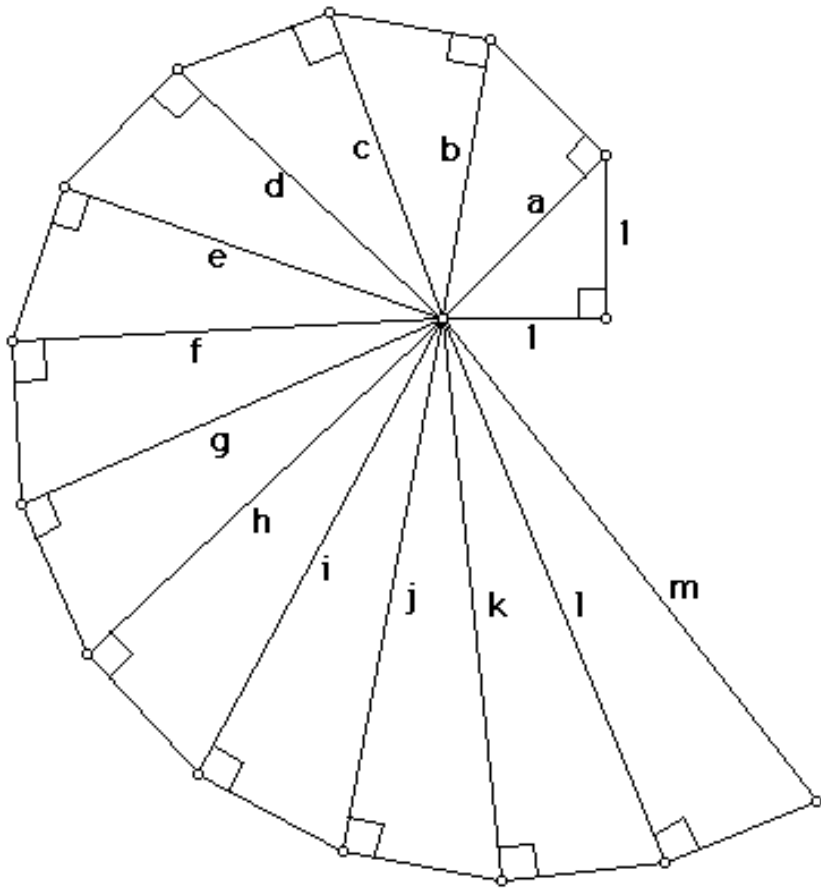
Adding these two equations gives

$$a^2 + b^2 = c(x + y) = c^2$$

Proof #4 (by President Garfield)

- In 1876, President Garfield discovered his own proof of the Pythagorean Theorem.
- The key is using the formula for the area of a trapezoid - *half sum of the bases times the altitude* - $(a+b)/2 \cdot (a+b)$.
- Looking at it another way, this can be computed as the sum of areas of the three triangles - $ab/2 + ab/2 + c \cdot c/2$.
- As before, simplifications yield $a^2 + b^2 = c^2$.





A Square Root Spiral



Pythagoras of Samos c. 570 - 500 B.C.E.

Distance Formula

- Also gave birth to the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Makes classical coordinate geometry “Euclidean”
 - If distance were measured some other way it would not be Euclidean geometry