Chapter 3

Greeks

Greece

- In 700 BC, Greece consisted of a collection of independent city-states covering a large area including modern day Greece, Turkey, and a multitude of Mediterranean islands.
- The Greeks were great travelers.
- Greek merchant ships sailed the seas, bringing them into contact with the civilizations of Egypt, Phoenicia, and Babylon.

Greece

- Also brought cultural influences like Egyptian geometry and Babylonian algebra and commercial arithmetic.
- Coinage in precious metals was invented around 700 BC and gave rise to a money economy based not only on agriculture but also on movable goods.

Greece

- This prosperous Greek society accumulated enough wealth to support a leisure class.
- Intellectuals and artists with enough time on their hands to study mathematics for its own sake, and generally, seeking knowledge for its own sake.
- They realized that non-practical activity is important in the advancement of knowledge.

The Greeks

- Made mathematics into one discipline.
- More profound, more rational, and more abstract (more remote from the uses of everyday life).
- In Egypt and Babylon, mathematics was a tool for practical applications or as special knowledge of a privileged class of scribes.

The Greeks

- Made mathematics a detached intellectual subject for the connoisseur instead of being monopolized by the powerful priesthood.
- They weren’t concerned with triangular fields, but with “triangles” and the characteristics of “triangularity.”
- The Greeks had a preference for the abstract.

The Greeks

- Best seen in the attitude toward the square root of 2.
  - The Babylonians computed it with high accuracy
  - The Greeks proved it was irrational
- Changed the nature of the subject of mathematics by applying reasoning to it $\Rightarrow$ **Proofs!**
  - Mathematical ‘truths’ must be proven!
  - Mathematics builds on itself.

“Proof by contradiction?”

- Ancient technique to prove a mathematical proposition
  - Assume something is true
    - E.g. “The square root of 2 is a rational number” $\frac{a}{b} = \sqrt{2}$
  - Follow through the logic
  - Find a contradiction
    - Thus prove that “The square root of 2 is **not** a rational number”
  - If the square root of 2 is rational, then there are integers a and b which are the smallest numbers for which no factors in common.

So we start with:

- Condition that integers **a** and **b** have ***<u>no</u>*** factors in common (except 1); and
- The assumption that

$$\frac{a}{b} = \sqrt{2}$$

- Now we square both sides to yield $\dfrac{a^2}{b^2} = 2$

- Rearrange to get $a^2 = 2b^2$
- Can now deduce that **a** must be an even number:
  - 2 times any integer (odd or even) is an even number
  - So we can express **a** as 2 times some other integer **c** $a = 2c$

- So **a** squared is: $a^2 = (2c)^2 = 4c^2$

- Now substitute this into equation for **a** squared above:

$$a^2 \left(= 4c^2\right) = 2b^2$$

- Divide last bit by 2 to yield $2c^2 = b^2$

- Which shows that b must also be even since 2 times any integer is an even number
- Therefore **b** is divisible by 2…
  - So **a** and **b** have 2 as a common factor!

“Proof by contradiction”

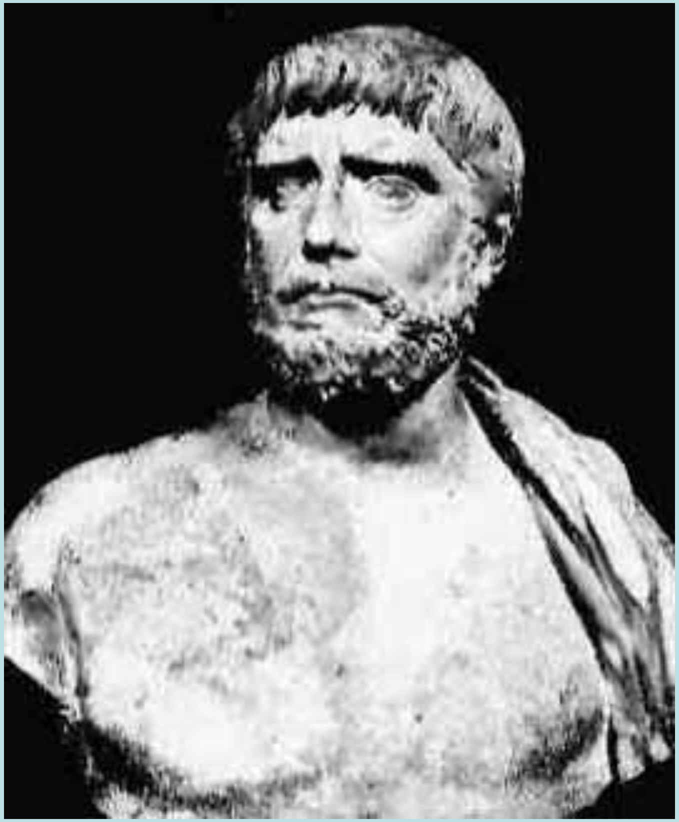
- But we began with the condition that **a** and **b** had no common factor—our assumption that the square root of 2 is rational has been contradicted by a series of logical steps.
- ***Therefore “proof by contradiction that the assumption that the square root of 2 is a rational number must be false***
- Therefore the square root of 2 must be *irrational*:
  - It cannot be equal to the ratio of two integers
- This is how Pythagoreans discovered irrational numbers
  - Didn’t like it—began with belief that all numbers were rational—but forced to accept it by logic

The Greeks

- Plato's inscription over the door of his academy, "*Let no man ignorant of geometry enter here.*"
- The Greeks believed that through inquiry and logic one could understand their place in the universe.
- The rise of Greek mathematics begins in the sixth century BC with Thales and Pythagoras.
- Later with Euclid, Archimedes, and Apollonius.
- Followed by Ptolemy, Pappus, and Diophantus.

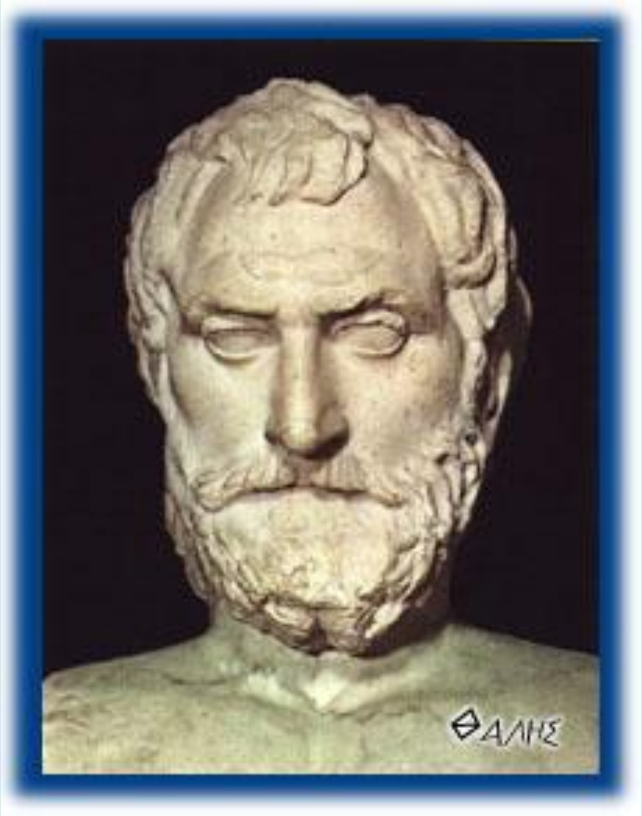
Thales of Miletus

- Born in Miletus, and lived from about 624 BC to about 547 BC.
- Thales was a merchant in his younger days, a statesman in his middle life, and a mathematician, astronomer, and philosopher in his later years.
- Extremely successful in his business ventures.



Thales of Miletus

- Thales used his skills to predict that the next season’s olive crop would be a very large one.
- He secured control of all the oil presses in Miletus and Chios in a year when olives promised to be plentiful, subletting them at his own rental when the season came.



Thales

- He was the first to predict a solar eclipse in 585BC.
- But, Historians often speculate that people already knew that eclipses occurred about every 19 years. Some say it was just a good guess.

Thales

- Traveled to Egypt, and probably Babylon, on commercial ventures, studying in those places and then bringing back the knowledge he learned about astronomy and geometry to Greece.
- He is hailed as the first to introduce using logical proof based on deductive reasoning rather than experiment and intuition to support an argument.

Thales

- Proclus states,
  - "*Thales was the first to go into Egypt and bring back this learning [geometry] into Greece. He discovered many propositions himself and he disclosed to his successors the underlying principles of many others, in some cases his methods being more general, in others more empirical.*"

Thales

- Founded the Ionian (Milesian) school of Greek astronomy.
- Considered the father of Greek astronomy, geometry, and arithmetic.
- Thales is designated as the first mathematician.
- The first of the Seven Sages of Greece.

Thales

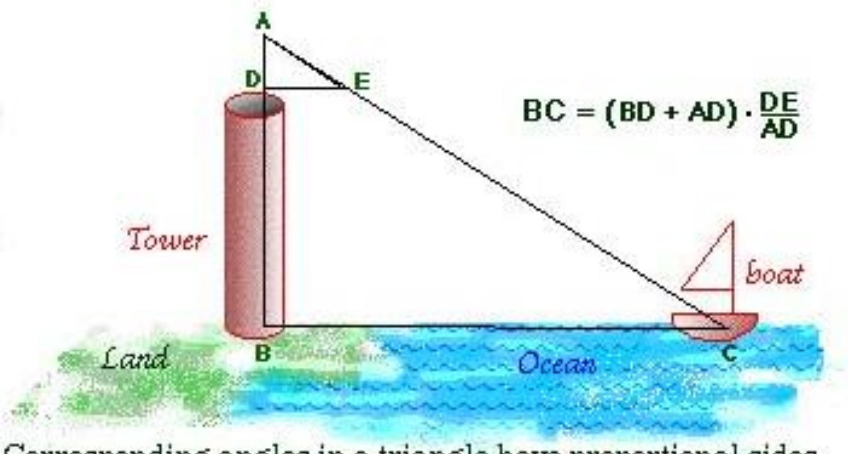
- His philosophy was that *“Water is the principle, or the element, of things. All things are water.”*
- He believed that the Earth floats on water and all things come to be from water.
- For him the Earth was a flat disc floating on an infinite ocean.
- Rough water would cause the earth quakes.
- Well, it is wrong, but, at least he believed that there was not a supernatural being that caused it.

Thales

- “Know thyself” and “nothing overmuch” were some of Thales philosophical ideas.
- Asked what was most difficult, he said, “To know thyself.”
- Asked what was easiest, he answered, “To give advice.”

Thales

- Thales measured the height of pyramids.
  - Thales discovered how to obtain the height of pyramids and all other similar objects, namely, by measuring the shadow of the object at the time when a body and its shadow are equal in length.
- Thales showed how to find the distances of ships from the shore necessarily involves the use of this theorem (iv).



Corresponding angles in a triangle have proportional sides.

Shadow Measurement

Thales is believed to have traveled to Mesopotamia (specifically, the city of Babylon) and to Egypt to acquire knowledge. While in Egypt, he learned to determine heights of tall objects by measuring the lengths of their shadows. Popular legend has it that Thales impressed the Egyptian ruler and priests by showing them how to find the height of the Great Pyramid of Cheops, but it seems more likely that the Egyptians taught Thales the technique (Hogben, 41). Here's how Thales might have done it (Burton, 86). At a certain time of day, the Great Pyramid cast a shadow of length 342 feet, as shown in Figure 1.

Figure 1

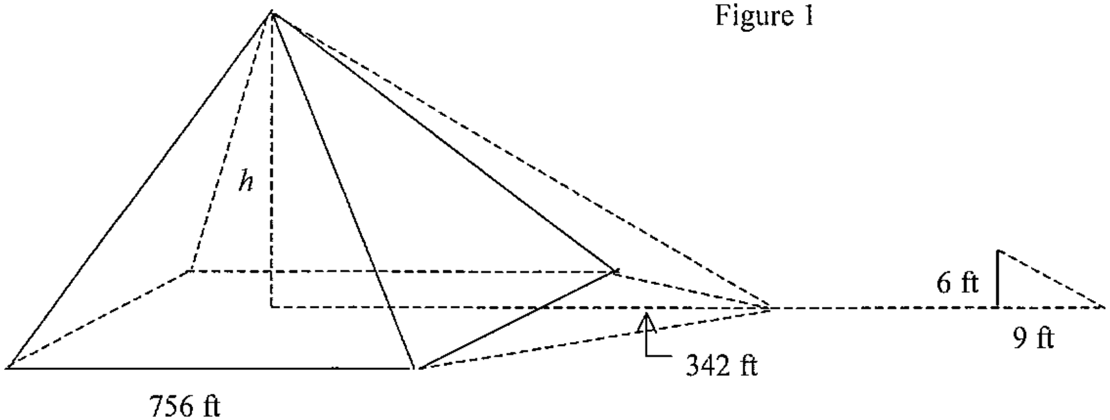
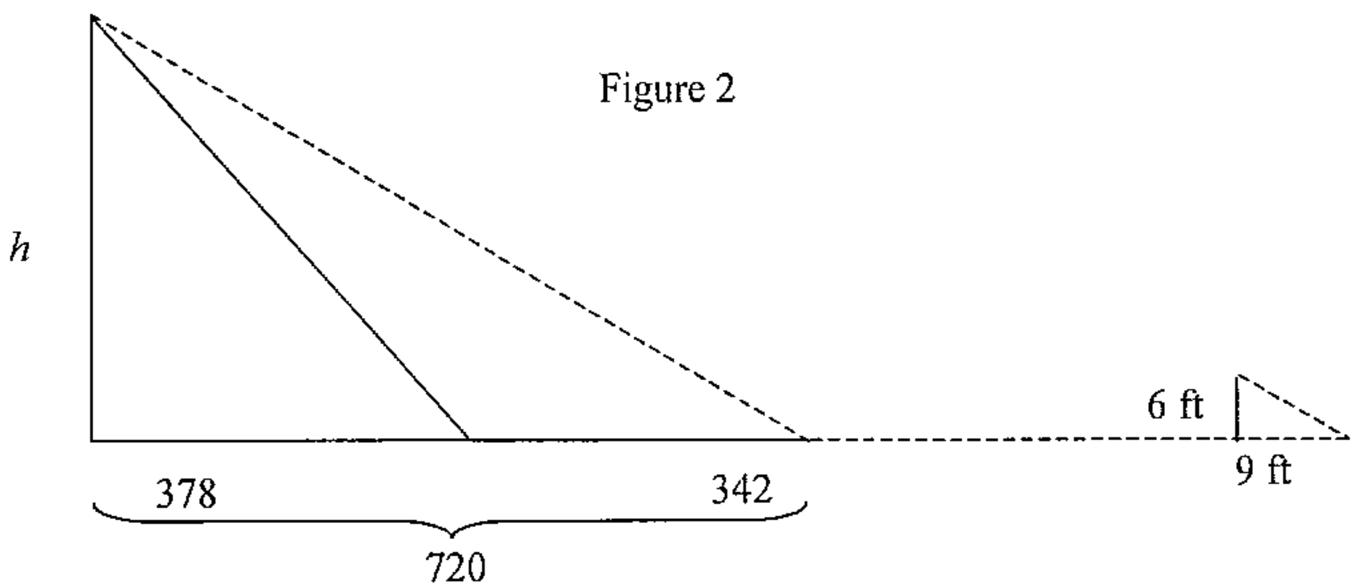


Figure 2

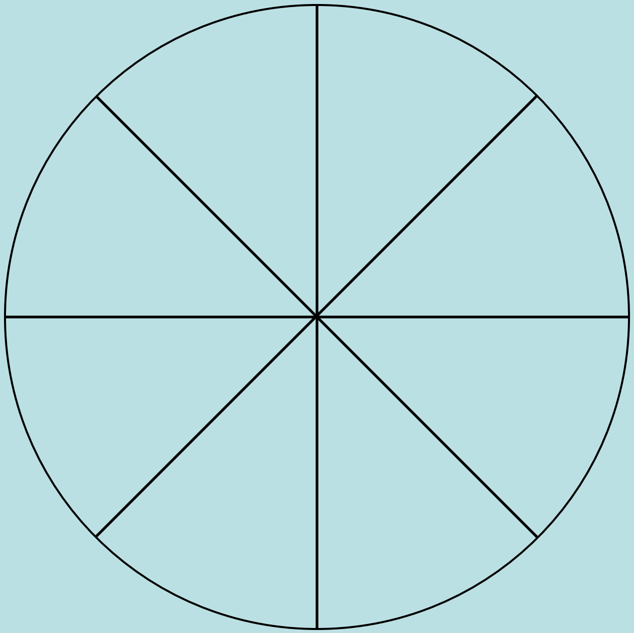


Thales

- Thales is credited with proving six propositions of elementary geometry:
  1. *A circle is bisected by its diameter.*
  2. *The base angles of an isosceles triangle are equal.*
  3. *If two straight lines intersect, the opposite angles are equal.*
  4. *Two triangles are congruent if they have one side and two adjacent angles equal.*
  5. *The sides of similar triangles are proportional.*
  6. *An angle inscribed in a semicircle is a right angle. (*)*

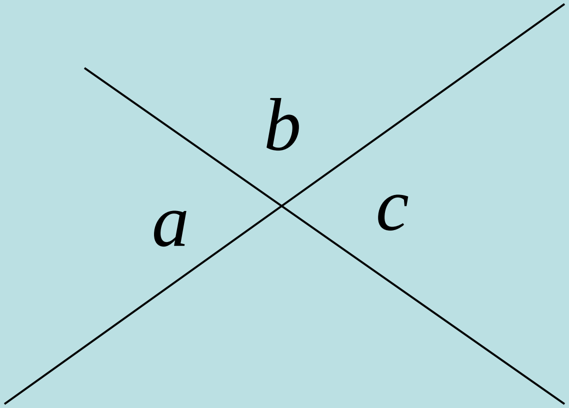
A circle is bisected by its diameter

- Thales supposedly **demonstrated** that a circle is bisected by its diameter.
- But Euclid did not even prove this, rather he only **stated** it.
- It seems likely that Thales also only stated it rather than **proving** it.



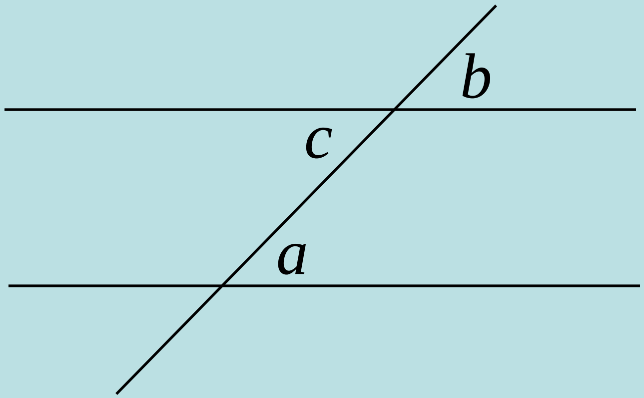
Vertical Angles Are Equal

- *The angles between two intersecting straight lines are equal.*
- $a + b = 180^{o} \Rightarrow a = 180^{o} - b$
- $b + c = 180^{o} \Rightarrow c = 180^{o} - b$
- $\therefore\ a = c.$



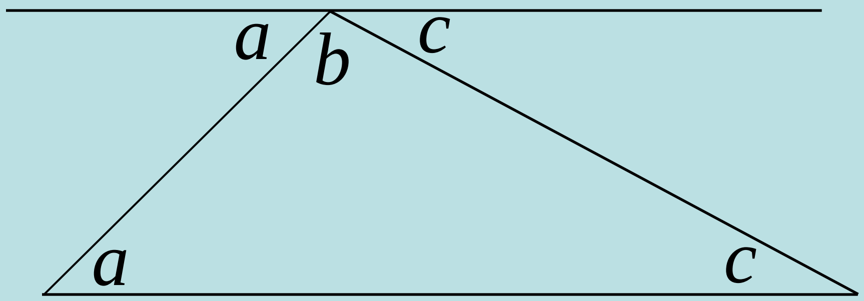
Alternate Interior Angles

- Thales immediately drew forth new truths from these six principles. He observed that a line crossing two given parallel lines makes equal angles with them.



Interior Angles in a Triangle

- The sum of the angles of any triangle is 180º.
- Draw a line through the upper vertex parallel to the base obtaining two pairs of alternate interior angles.
- $\therefore\ a + b + c = 180^{\circ}$.



Thales

- *An angle in a semicircle is a right angle.*

$$\alpha + \beta = 180°$$

$$2\gamma + \alpha = 180°$$

$$2\delta + \beta = 180°$$

$$2\delta + \beta + 2\gamma + \alpha = 360°$$

$$2(\delta + \gamma) + (\alpha + \beta) = 360°$$

$$-(\alpha + \beta) = -180°$$

$$2(\delta + \gamma) = 180°$$

$$\therefore \ \delta + \gamma = 90°.$$