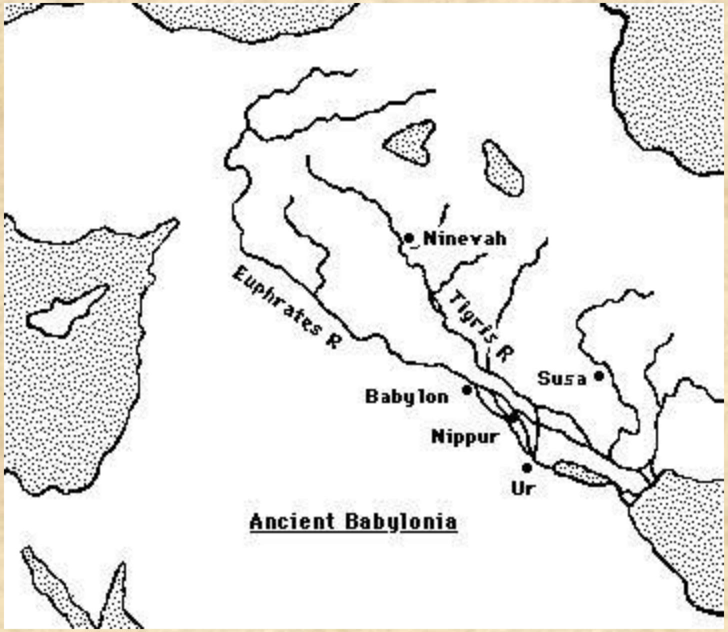
Babylonian Mathematics

1900 BC and 1600 BC

Babylonian near IRAQ. (Between two rivers)



Cradle of Civilization

Babylonians

- While peasants, priests and civil servants used mathematics in Egyptian times.
- Merchants used mathematics in Babylonian times.
- Egyptian were know for their Geometry , but Babylonians known for Algebra. (not the algebra we see in school, no symbols; but words and algorithms)
- More advanced than Egyptian mathematics.
- Could find square and cube roots.
- Worked with Pythagorean triples 1200 years before Pythagoras.
- had a knowledge of pi and possibly e.
- solve some quadratics and even polynomials of degree 8.
- tended to think algorithmically; that is, in terms of a sequence of steps.

Babylonians

- Concentrated more on algebra and less on geometry, in contrast to the Greeks.
- The Babylonians were aware of the link between algebra and geometry.
- They used terms like length and area in their solutions of problems.
- They had no objection to combining lengths and areas, thus mixing dimensions.
- tended to think algorithmically; that is, in terms of a sequence of steps.
- did not attempt any formal proof.
- They used base 60.

Why 60?

- Theon's answer was that 60 was the smallest number divisible by 1, 2, 3, 4, and 5 so the number of divisors was maximized. But this is a little too high level, Why not 12? Divisors 1,2,3,4.

Why 60?

- Several theories have been based on astronomical events. The suggestion that 60 is the product of the number of months in the year ((12) moons per year) with the number of planets ((5)Mercury, Venus, Mars, Jupiter, Saturn) again seems far fetched as a reason for base 60 .

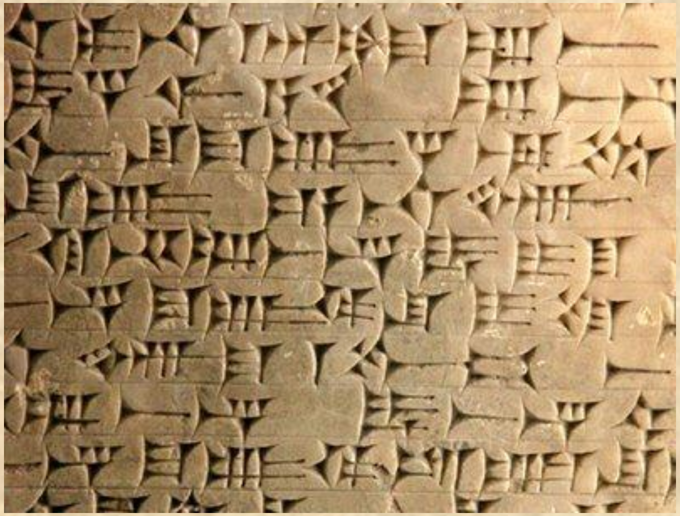
Why 60?

- Equilateral triangle was considered the fundamental geometrical building block by the Sumerians. Now an angle of an equilateral triangle is 60 so if this were divided into 10, an angle of 6 would become the basic angular unit. Now there are sixty of these basic units in a circle so again we have the proposed reason for choosing 60 as a base.
- Some said it was a combination of two civilizations one group using base ten and the other group using base 6.

It works?.....It is still around!

- 60 minutes in an hour.
- 60 seconds in a minute.
- 360 degrees in a circle.
- 24 hour clock is from the ancient Babylonians.
- The used time for a measure of distance.
- Historians said they could walk 12 miles a day. That is where hours came from.

Tables



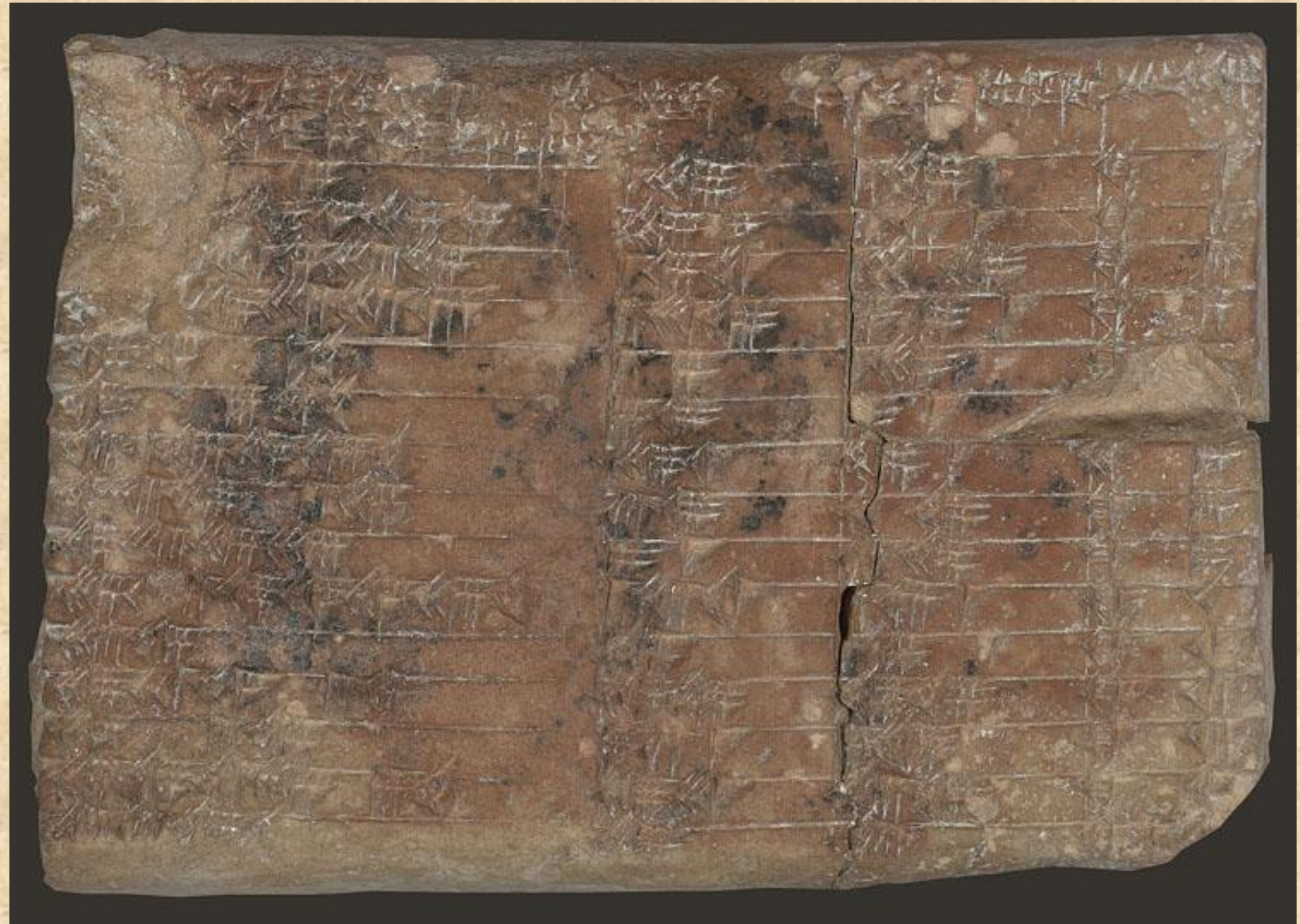
- Multiplication tables
- Reciprocal tables
- Tables of squares
- Table of cubes
- Square roots
- Cube roots
- Some powers
- Coefficient lists –conversion factors for weights & measures

Tablets

- Because the Latin word for “wedge” is *cuneus*, the Babylonian writing on clay tablets using a wedge-shaped stylus is called **cuneiform**.
- Originally, deciphered by a German schoolteacher Georg Friedrich Grotefend (1775-1853) as a drunken wager with friends.
- Later, re-deciphered by H.C. Rawlinson (1810-1895) in 1847.
- Over 300 tablets have been found containing mathematics.

Plimpton 322

Catalog number 322 in the G. A. Plimpton Collection at Columbia University



Plimpton 322

Width	Diagonal	
119	169	1
3367	4825(11521)	2
4601	6649	3
12709	18541	4
65	97	5
319	481	6
2291	3541	7
799	1249	8
481(541)	769	9
4961	8161	10
45	75	11
1679	2929	12
161(25921)	289	13
1771	3229	14
56	106(53)	15

- It consists of fifteen rows and four columns.
- Let’s look at the three on the right.
- The far right is simply the numbering of the lines.

Plimpton 322

Width	Diagonal	
119	169	1
3367	4825(11521)	2
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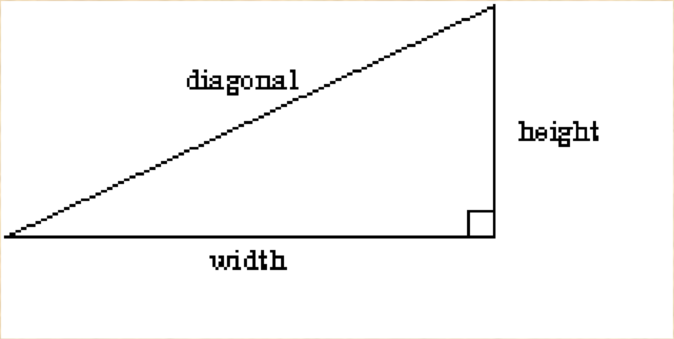
161(25921)	289	13
1771	3229	14
56	106(53)	15

- The next two columns, with four exceptions, are the hypotenuse and one leg of integral sided right triangles.
- The four exceptions are shown with the original number in parentheses.



Plimpton 322

- The fourth column gives the values of $(c/a)^2$.
- These values are the squares of the *secant* of angle *B* in the triangle.
- This makes the tablet the oldest record of trigonometric functions.
- It is a secant table for angles between 30° and 45°.

$(119/120)^2$	119	169	1
$(3367/3456)^2$	3367	4825	2
$(4601/4800)^2$	4601	6649	3
$(12709/13500)^2$	12709	18541	4
$(65/72)^2$	65	97	5
$(319/360)^2$	319	481	6
$(2291/2700)^2$	2291	3541	7
$(799/960)^2$	799	1249	8
$(481/600)^2$	481	769	9
$(4961/6480)^2$	4961	8161	10
$(3/4)^2$	45	75	11
$(1679/2400)^2$	1679	2929	12
$(161/240)^2$	161	289	13
$(1771/2700)^2$	1771	3229	14
$(28/45)^2$	56	106	15

Perhaps the most amazing aspect of the Babylonian's calculating skills was their construction of tables to aid calculation. Two tablets found at Senkerah on the Euphrates in 1854 date from 2000 BC. They give squares of the numbers up to 59 and cubes of the numbers up to 32.

The Babylonians used the formula
$ab = [(a + b)^2 - a^2 - b^2]/2$
to make multiplication easier.

Even better is their formula
$ab = [(a + b)^2 - (a - b)^2]/4$

- Using tablets containing squares, the Babylonians could use the formula

$$ab = [(a+b)^2 - a^2 - b^2] \div 2$$

- Or, an even better one is

$$ab = [(a+b)^2 - (a-b)^2] \div 4$$

$$ab = [(a+b)^2 - (a-b)^2] \div 4$$

10	100	19	361
11	121	20	400
12	144	21	441
13	169	22	484
14	196	23	529
15	225	24	576
16	256	25	625
17	289	26	676
18	324	27	729

- Using the table at the right, find 11×12.
- Following the formula, we have $11 \times 12 =$

  $(23^2 - 1^2) \div 4 = 121$

Why/How does this work?

$$ab = [(a+b)^2 - a^2 - b^2] \div 2$$

Cuneiform

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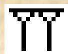				
				
				
				
				
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			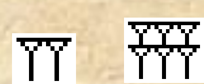	
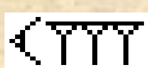				

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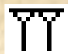	2			
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			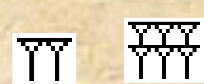	
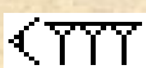				

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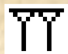
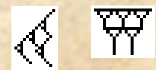
	2			
	3			
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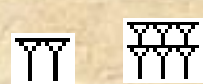
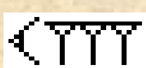
	10			
	13			

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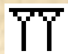	2			
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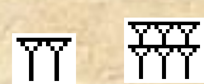
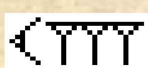
	10			
	13			

Column I	Value		Column I	Value
𒐕	1		𒎙𒁹𒇲	19
𒐖	2		𒎙	20
𒐗	3		𒐏 𒐙	45
𒐘	4			
𒐙	5		𒐕 𒐗	
𒐚	6		𒐕 𒌍𒐝	
. . .				
𒌋	10		𒐖 𒐚	
𒌋𒐗	13			

Column I	Value		Column I	Value	
𒁹	1		𒌋𒌋𒁹𒇲	19	
𒈫	2		𒌋𒌋	20	
𒐈	3		𒐏 𒐊	45	
𒐉	4				
𒐊	5		𒁹 𒐈	63	**1,3**
𒐋	6		𒁹 𒌍𒐍		
. . .					
𒌋	10		𒈫 𒐋		
𒌋𒐈	13				

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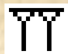	2			20	
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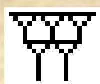	5			63	**1,3**
	6			99	**1,39**
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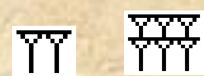
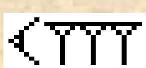
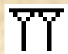
	10				
	13				

Column I	Value		Column I	Value	
	1			19	
	2			20	
	3			45	
	4				
	5			63	**1,3**
	6			99	**1,39**
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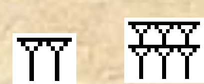
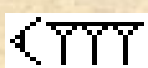
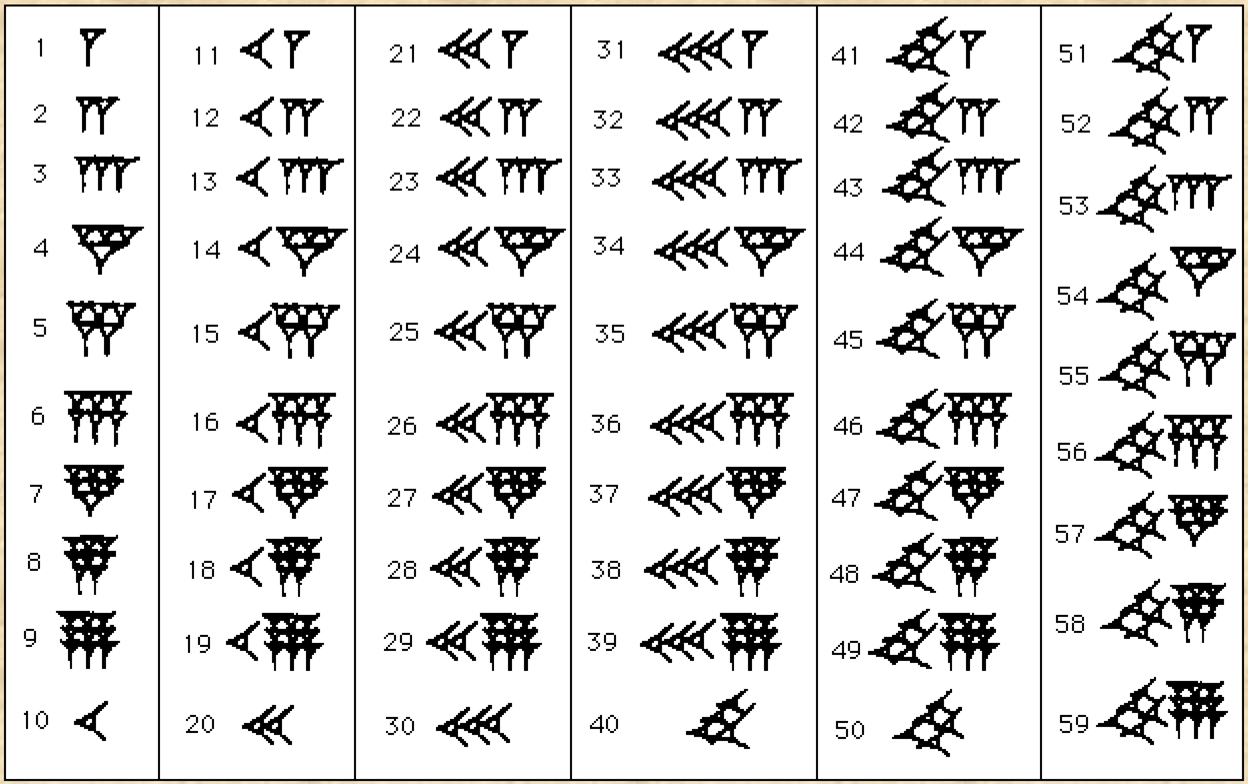
	10			126	**2,6**
	13				

Babylonian Number System



Base 10

12345

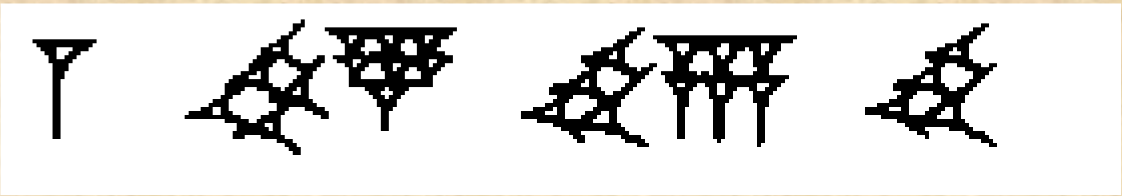
$$1\times10^4+2\times10^3+3\times10^2+4\times10^1+5\times10^0$$

12.345

$$10+2+\frac{3}{10}+\frac{4}{100}+\frac{5}{1000}$$

$$1\bullet10^1+2\bullet10^0+3\bullet10^{-1}+4\bullet10^{-2}+5\bullet10^{-3}$$

Base 60

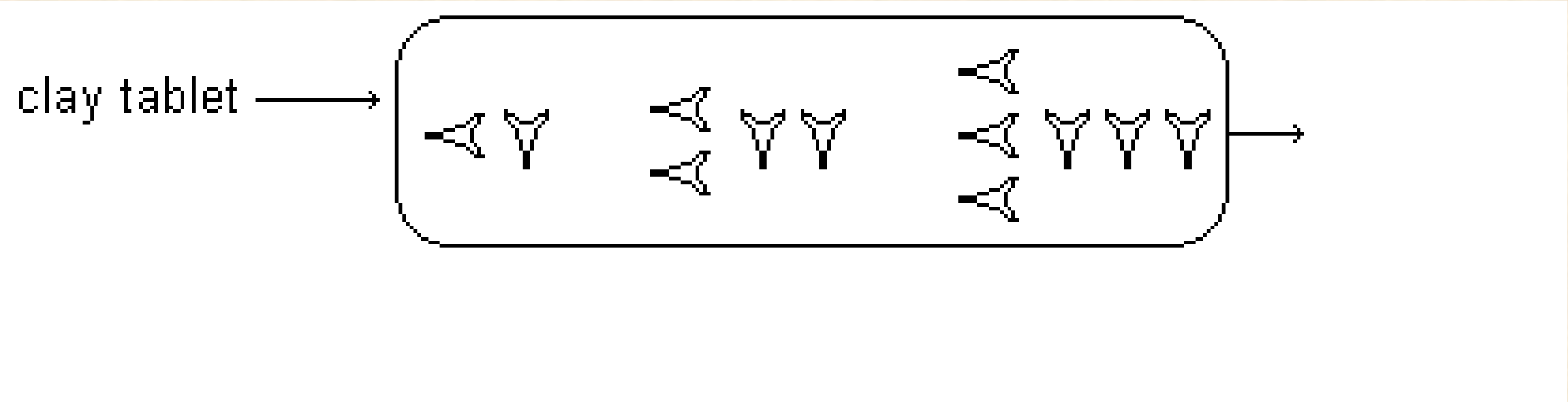


1,57,46,40

$$1\times 60^3+57\times 60^2,46\times 60^1,40\times 60^0$$

216000+205200+2760+40

424,000



What is this number?

11,22,33

1,25,30 in the **sexagesimal** system is

$1 \times 60^2 + 25 \times 60 + 30$

$= 3600 + 1500 + 30$

$= 5{,}130$ in our base 10 system

- 2,18,6,59 in the **sexagesimal** system is

- $2 \times 60^3 + 18 \times 60^2 + 6 \times 60 + 59$

  $= 432{,}000 + 64{,}800 + 360 + 59$

  $= 497{,}219$ in our base 10 system

PROBLEMS!?!?

- 1,25,30 in the **sexagesimal** system is
- $1\times60^2 + 25\times60 + 30 = 5{,}130$

But it could be

$1\times60^3 + 25\times60^2 + 30\times60 = 307{,}800$

or

$1\times60^4 + 25\times60^3 + 30\times60^2 = 18{,}468{,}000$

- The Babylonians <u>did</u> use a sign for zero
- But only to denote an empty space **<u>inside</u>** a number

- e.g. to distinguish
  - 1,0,30 = 3630 from
  - 1,30 = 90

Let's Try some other bases

$$1234_5$$

$$1 \bullet 5^3 + 2 \bullet 5^2 + 3 \bullet 5^1 + 4 \bullet 5^0$$

$$1 \bullet 125 + 2 \bullet 25 + 3 \bullet 5 + 4 \bullet 1$$

$$125 + 50 + 15 + 4 = 194$$

- Each base b will have exactly b digits. Binary has two digits 0 and 1.
- If the base is larger than 10, we use the letters A,B,C,D,E…etc to represent 10, 11,12,13,….
- The exception is base twelve (duodecimal) where X represent 10 and E represent 11.

Let’s work in the other direction

Convert 198 base 10 to base 5.

Think, base 5 ----5, 25,125,625

625 doesn’t go into 198,

125 goes into 198, 1 time, with remainder 73.

25 goes into 73, 2 times, with remainder 23

5 goes into 23, 4 times, with remainder 3

$$1\bullet 5^3 + 2\bullet 5^2 + 4\bullet 5^1 + 3\bullet 5^0$$

$$1243_5$$

- Convert our (base 10) 4,137 to sexagesimal:

- $4{,}137 = \underline{1}\times 3{,}600+537 = \underline{1}\times 60^2+537$
- $537= \underline{8}\times 60+\underline{57}$
- $4{,}137= \underline{1}\times 60^2+\underline{8}x60+\underline{57}$ = **1,8,57**

- Convert 11,944 to sexagesimal:

$11{,}944 = \underline{3} \times 3{,}600 + 1{,}144$

$1{,}144 = \underline{19} \times 60 + \underline{4}$

$11{,}944 = \underline{3} \times 60^2 + \underline{19} x 60 + \underline{4}$

= **3,19,4**

Let's try some more

46_7

101_2

268_9

$XE2_{12}$

Let's try some more

$46_7 = 34$

$101_2 = 5$

$268_9 = 224$

$XE2_{12} = 1574$

1945

- In 1945, when Neugebauer and A. Sachs published the translation of cuneiform tablet YBC 7289 from the Yale collection.
- This is when we learned that ancient Babylon (1800-1600 B.C.) possessed a base-60 formula for the square root of 2 accurate to five decimal places (1.41421+)
- The formula for generating all Pythagorean triples (a triangle with sides of 3, 4, and 5 units is merely an example) a thousand years before Pythagoras.

Pythagorean Theorem

A translation of a Babylonian tablet which is preserved in the British museum goes as follows:

4 *is the length and* 5 *the diagonal. What is the breadth* ?
Its size is not known.
4 *times* 4 *is* 16.
5 *times* 5 *is* 25.
You take 16 *from 25 and there remains* 9.
What times what shall I take in order to get 9 ?
3 *times* 3 *is* 9.
3 *is the breadth.*

Solve the following simultaneously $x+y=a, xy=b$

$$\frac{x+y}{2}=\frac{a}{2}$$

$$\left(\frac{x+y}{2}\right)^2=\left(\frac{a}{2}\right)^2$$

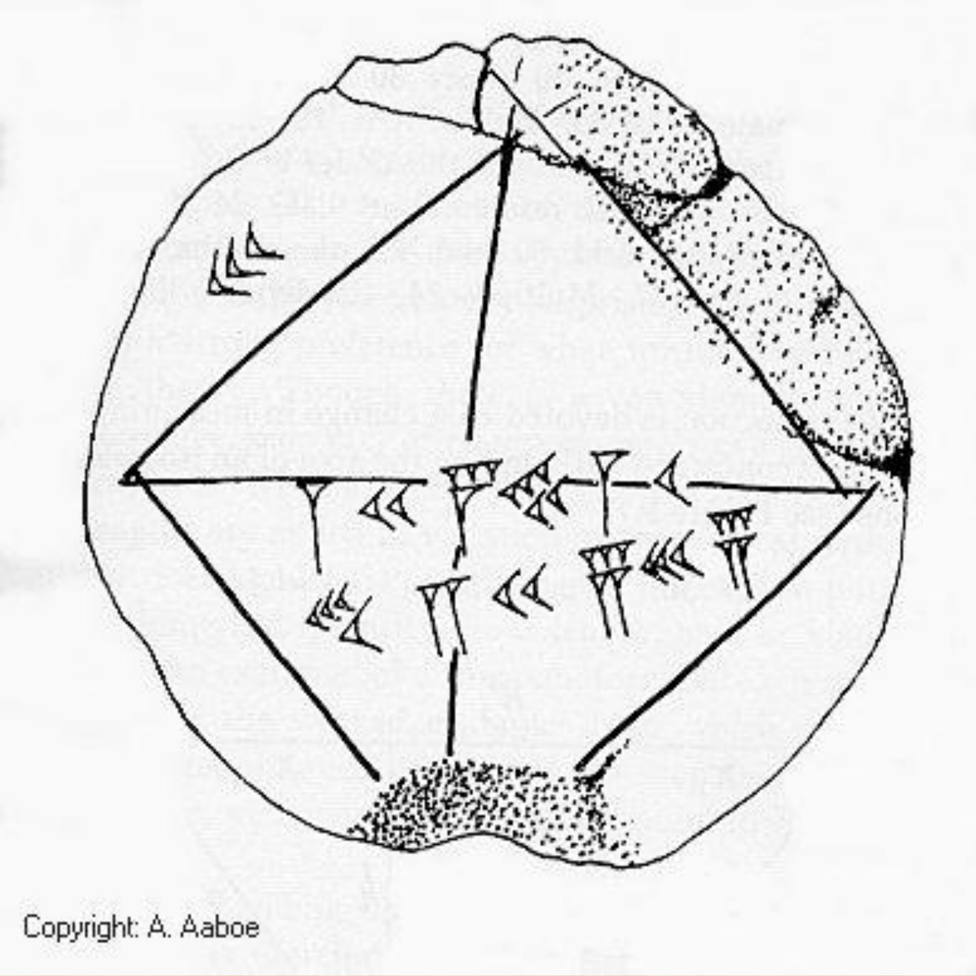
$$\left(\frac{x+y}{2}\right)^2-xy=\left(\frac{a}{2}\right)^2-b$$

$$\frac{x^2+2xy+y^2}{4}=\frac{a^2}{4}-b$$

$$\frac{x+y}{2}=\pm\sqrt{\frac{a^2}{4}-b}$$

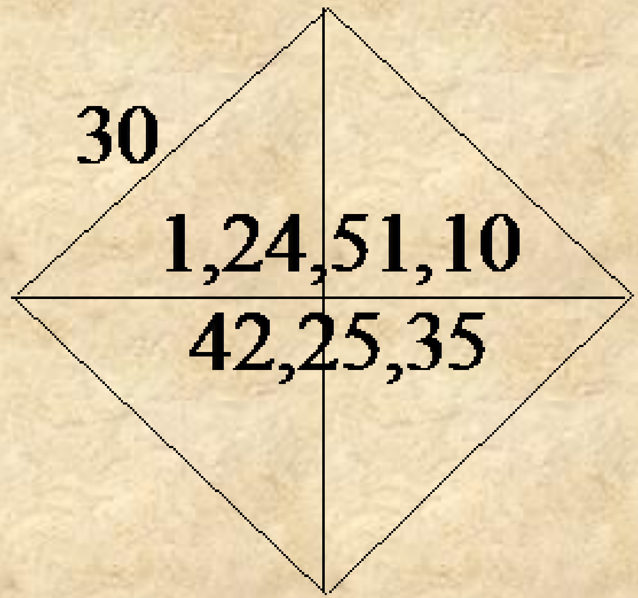
- It shows evidence of square roots, and solving equations.
- They usually would solve for some specific numbers, and they didn't use our notations.
- They solved quadratics, all types if they had at least one positive root. The problems we teach from 10^{th} grade textbooks originated 4000 years ago.

Tablet YBC 7289 from the Yale collection (1800 B.C)



What does this mean?

- They found the diagonal of a square with side 30.
- Well, how close do you the Babylonians got?
- 42,25,35 should have been 42;25,35 (remember base 60)



$$(42\times 60^{0})+(25\times 60^{-1})+(35\times 60^{-2})$$

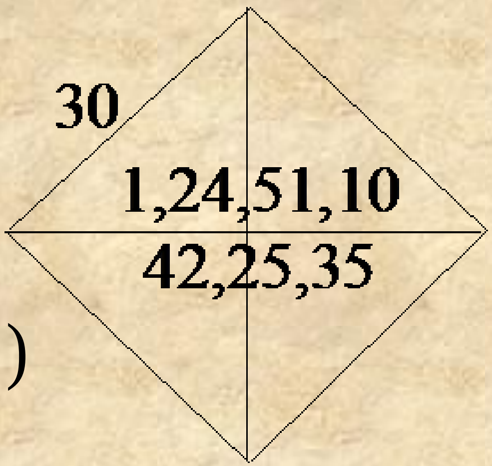
$$(42)+(\frac{25}{60})+(\frac{35}{3600})$$

42.42638

•How does that compare to our approximation?

42.4264

What about the other number?



1,24,51,10?

Probably 1;24,51,10

$$(1\times 60^{0})+(24\times 60^{-1})+(50\times 60^{-2})+(10\times 60^{-3})$$

$$(1)+(\frac{24}{60})+(\frac{50}{3600})+(\frac{10}{216000})$$

$\sqrt{2} = 1.414213562$

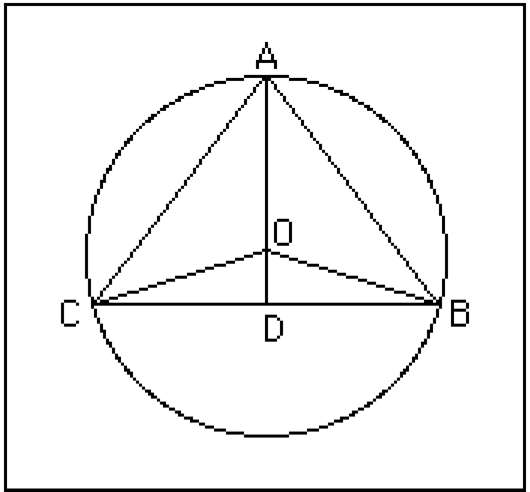
1.414212963….what is that?

$\sqrt{2}$?

WOW

Susa Tablet

The Susa tablet sets out a problem about an isosceles triangle with sides 50, 50 and 60. The problem is to find the radius of the circle through the three vertices.



The Susa tablet

- Here we have labeled the triangle *A*, *B*, *C* and the centre of the circle is *O*. The perpendicular *AD* is drawn from *A* to meet the side *BC*. Now the triangle *ABD* is a right angled triangle so, using Pythagoras's theorem $AD^2 = AB^2 - BD^2$, so *AD* = 40.
- Let the radius of the circle by *x*. Then *AO* = *OB* = *x* and *OD* = 40 - *x*.
- Using Pythagoras's theorem again on the triangle *OBD* we have

  $x^2 = OD^2 + DB^2$.
- So

  $x^2 = (40-x)^2 + 30^2$
- giving $x^2 = 40^2 - 80x + x^2 + 30^2$
- and so $80x = 2500$ or, in sexagesimal, *x* = 31;15.

http://www.sciencephoto.com/media/82992/enlarge