Egyptian Method of False Position

Egyptian Scribe Ahmes

Rhind papyrus approx1659 BCE

In particular, the Papyrus has a series of problems showing how to solve what we could call 'linear equations' by a method that became commonly used by merchants throughout the Mediterranean countries for some three thousand years, called the 'Method of False Position'.

It is used for all kinds of calculations involving comparison of values and quantities. This method still appeared in school text books in the early 20th century.

- False position begins by selecting a convenient answer or making an educated guess, one that makes the calculations of the problem simpler.
- The guess does not have to be the correct answer.
- After calculating the result from the convenient answer, a false position problem is solved by using the result to determine how to adjust the convenient answer to make it correct.

This belongs to the group of 'aha' problems, sometimes written as 'heap' meaning 'quantity' or 'number'. $\bar{4}$ is an example of the usual way we represent Egyptian 'unit' fractions.

PROBLEM #26 : In a quantity and its $\bar{4}$ (is added) to it so that 15 results. A quantity is added to a quarter of itself and the answer is 15

$$x + \frac{1}{4}x = 15$$

Make a convenient guess at the solution….It doesn't have to be correct….

Let's pick 4…

- Why?…
- It simplifies the calculation…$x=4$

$$x+\frac{1}{4}x=15 \qquad (4)+\frac{1}{4}(4)=15 \qquad \begin{gathered}(4)+1\neq 15\\ 5\neq 15\end{gathered}$$

- What multiple is our "guess" off? $\frac{15}{5}=3$
- Take our "guess" and multiply by multiple.
- Answer $3\bullet 4=12$
- Check

$$x+\frac{1}{4}x=15 \qquad (12)+\frac{1}{4}(12)=15$$

‘Aha, its seventh, it makes 19’

- What is the translation?
- What is our guess??? $x = 7$

$$x + \frac{1}{7}x = 19 \qquad (7) + \frac{1}{7}(7) = 19 \qquad \begin{array}{l}(7) + 1 \neq 19 \\ 8 \neq 19\end{array}$$

- What multiple is our “guess” off? $\frac{19}{8}$
- Take our “guess” and multiply by multiple.
- Answer $\frac{19}{8} \bullet 7 = \frac{133}{8}$
- Check

$$x + \frac{1}{7}x = 19 \quad \left(\frac{133}{8}\right) + \frac{1}{7}\left(\frac{133}{8}\right) = 19 \quad \left(\frac{133}{8}\right) + \left(\frac{133}{56}\right) = 19 \quad \left(\frac{931}{56}\right) + \left(\frac{133}{56}\right) = 19 \quad \left(\frac{1064}{56}\right) = 19$$

A quantity and its half are added to become 16.

- What is the translation?
- What is our guess??? $x = 2$

$$x + \frac{1}{2}x = 16 \qquad (2) + \frac{1}{2}(2) = 16 \qquad \begin{aligned}(2) + 1 &\neq 16 \\ 3 &\neq 16\end{aligned}$$

- What multiple is our "guess" off? $\frac{16}{3}$
- Take our "guess" and multiply by multiple.
- Answer $\frac{16}{3} \bullet 2 = \frac{32}{3}$
- Check

$$x + \frac{1}{2}x = 16 \quad \left(\frac{32}{3}\right) + \frac{1}{2}\left(\frac{32}{3}\right) = 16 \quad \left(\frac{32}{3}\right) + \left(\frac{32}{6}\right) = 16 \quad \left(\frac{64}{6}\right) + \left(\frac{32}{6}\right) = 16 \quad \left(\frac{96}{6}\right) = 16$$

A quantity and its fifth are added to become 21.

- What is the translation?
- What is our guess??? $x = 5$

$$x + \frac{1}{5}x = 21 \qquad (5) + \frac{1}{5}(5) = 21 \qquad \begin{matrix} (5) + 1 \neq 21 \\ 6 \neq 21 \end{matrix}$$

- What multiple is our "guess" off? $\frac{21}{6}$
- Take our "guess" and multiply by multiple.
- Answer $\frac{21}{6} \bullet 5 = \frac{105}{6}$
- Check

$$x + \frac{1}{5}x = 21 \left(\frac{105}{6}\right) + \frac{1}{5}\left(\frac{105}{6}\right) = 21 \quad \left(\frac{105}{6}\right) + \left(\frac{105}{30}\right) = 21 \quad \left(\frac{525}{30}\right) + \left(\frac{105}{30}\right) = 21 \quad \left(\frac{630}{30}\right) = 21$$

A quantity and its third and its fourth are added to become 2.

- What is the translation?
- What is our guess??? $x = 12$

$x + \frac{1}{3}x + \frac{1}{4}x = 2$ $\quad (12) + \frac{1}{3}(12) + \frac{1}{4}(12) = 2$ $\quad 12 + 4 + 3 \neq 2$

$19 \neq 2$

- What multiple is our "guess" off? $\frac{2}{19}$
- Take our "guess" and multiply by multiple.
- Answer $\frac{2}{19} \bullet 12 = \frac{24}{19}$
- Check

$x + \frac{1}{3}x + \frac{1}{4}x = 2$ $\quad \left(\frac{24}{19}\right) + \frac{1}{3}\left(\frac{24}{19}\right) + \frac{1}{4}\left(\frac{24}{19}\right) = 2$ $\quad \left(\frac{24}{19}\right) + \left(\frac{8}{19}\right) + \left(\frac{6}{19}\right) = 2$ $\quad \left(\frac{38}{19}\right) = 2$

A quantity and its two-thirds, its half and its seventh are added to become 2.

- What is the translation?
- What is our guess??? $x = 42$

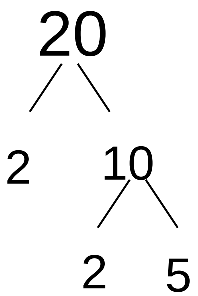
$$x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 2 \quad 42 + \frac{2}{3}(42) + \frac{1}{2}(42) + \frac{1}{7}(42) = 2 \quad \begin{aligned} &42 + 28 + 21 + 6 \neq 2 \\ &97 \neq 2 \end{aligned}$$

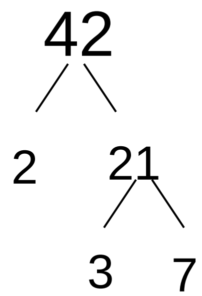
- What multiple is our "guess" off? $\frac{2}{97}$
- Take our "guess" and multiply by multiple.
- Answer $\frac{2}{97} \bullet 42 = \frac{84}{97}$
- Check

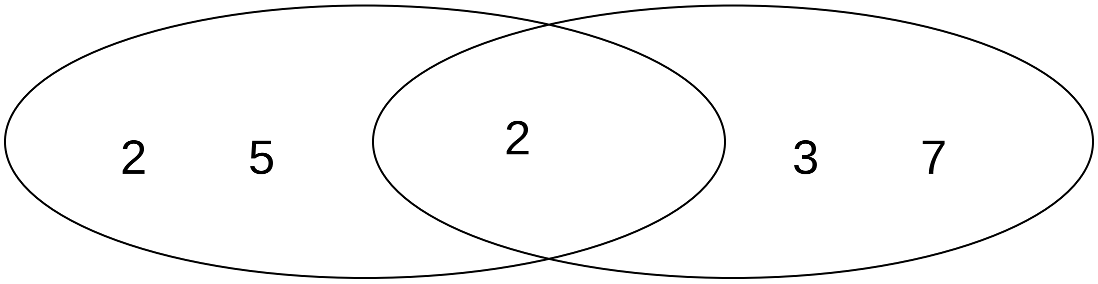
$$x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 2 \quad \frac{84}{97} + \frac{2}{3}\left(\frac{84}{97}\right) + \frac{1}{2}\left(\frac{84}{97}\right) + \frac{1}{7}\left(\frac{84}{97}\right) = 2 \quad \frac{84}{97} + \left(\frac{56}{97}\right) + \left(\frac{42}{97}\right) + \left(\frac{12}{97}\right) = 2 \quad \left(\frac{194}{19}\right) = 2$$

Prime Factorization

Make a factor tree.







LCM= $2 \cdot 5 \cdot 2 \cdot 3 \cdot 7 = 420$

Make factor trees for 45,60,96, write down their prime factors and determine their LCM.

45 $= 3^2 \times 5$

60 $= 2^2 \times 3 \times 5$

96 $= 2^5 \times 3$

The LCM = $2^5 \times 3^2 \times 5 = 1440$

Use the method of false position and this information and to solve

$$x + \frac{2}{45}x + \frac{13}{60}x + \frac{29}{96}x = 20$$

$$x+\frac{2}{45}x+\frac{13}{60}x+\frac{29}{96}x=20$$

What is our guess??? $x=1440$

$$x+\frac{2}{45}x+\frac{13}{60}x+\frac{29}{96}x=20$$

$$1440+\frac{2}{45}(1440)+\frac{13}{60}(1440)+\frac{29}{96}(1440)=20$$

$$1440+64+312+435\neq 20$$

$$2251\neq 20$$

- What multiple is our "guess" off? $\frac{20}{2251}$
- Take our "guess" and multiply by multiple.
- Answer $\frac{20}{2251}\bullet 1440=\frac{28800}{2251}$
- Check

$$x+\frac{2}{45}x+\frac{13}{60}x+\frac{29}{96}x=20$$

$$\left(\frac{28800}{2251}\right)+\frac{2}{45}\left(\frac{28800}{2251}\right)+\frac{13}{60}\left(\frac{28800}{2251}\right)+\frac{29}{96}\left(\frac{28800}{2251}\right)=20$$

$$\left(\frac{28800}{2251}\right)+\left(\frac{1280}{2251}\right)+\left(\frac{6240}{2251}\right)+\left(\frac{8700}{2251}\right)=20$$

$$\left(\frac{45020}{2251}\right)=20$$