

# Egyptian Multiplication

We have to memorize multiplication tables. Egyptian did not.

# Egyptian Multiplication

- Unique method which they correctly viewed as repeated addition.
- Based on doubling and is also known as *the didactic method*.
- Starting with one and doubling, they obtained a never-ending sequence of numbers: 1, 2, 4, 8, 16, 32, 64, 128, ...
- These numbers are the powers of two:  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$ ,  $2^4$ ,  $2^5$ ,  $2^6$ ,  $2^7$ , ...

# Egyptian Multiplication

- Egyptians figured out is that any integer can be written as a sum of the powers of two without repeating any of them
  - For example,
    - $11 = 8 + 2 + 1$
- Try 23, 44 and 158
- $23 = 16 + 4 + 2 + 1$
  - $44 = 32 + 8 + 4$
  - $158 = 128 + 16 + 8 + 4 + 2$

# Egyptian Multiplication

- Suppose we want to multiply  $12 \times 17$ .
- Start with 1 and 17.
- Keep doubling both numbers until the right side gets as close as possible to, but not larger than 12.

17		1
34		2
68		4
136		8

# Egyptian Multiplication

- We want the powers of 2 that add to give us 12.
- We want 12 because we are multiplying  
17 x 12

17		1
34		2
*68		4
*136		8

# Egyptian Multiplication

- To obtain the answer, add the corresponding left side numbers of the starred positions.

17		1
34		2
*68		4
*136		8

- In this case,
  - ✓  $136 + 68 = 204$
- So,  $12 \times 17 = 204$ .

# Why it works?

- This ingenious method relies on the distributive law  $a \times (b + c) = a \times b + a \times c$
- Since  $12 = 4 + 8$ , we can write
$$17 \times 12 = 17 \times (4 + 8) = 17 \times 4 + 17 \times 8 = 68 + 136 = 204$$
- Not bad for thousands of years ago!

Let's multiply 49 and 61 together.

$$61 \mid 1 *$$

$$61+61=122 \mid 2$$

$$122+122=244 \mid 4$$

$$244+244=488 \mid 8$$

$$488+488=976 \mid 16 *$$

$$976+976=1952 \mid 32 *$$

What numbers combine to give us 49?

$$\text{Now } = (32+16+1) \cdot 61$$

$$\text{Which is } = (32 \cdot 61) + (16 \cdot 61) + (1 \cdot 61)$$

$$= 1952 + 976 + 61$$

$$= 2989$$



Try this method and multiply 36 and 23 together.

$$36 \mid 1 *$$

$$36+36=72 \mid 2 *$$

$$72+72=144 \mid 4 *$$

$$144+144=288 \mid 8$$

$$288+288=576 \mid 16 *$$

What numbers combine to give us 23?

$$\text{Now } = (16+4+2+1) \cdot 36$$

$$\text{Which is } = (16 \cdot 36) + (4 \cdot 36) + (2 \cdot 36) + (1 \cdot 36)$$

$$= 576 + 144 + 72 + 36$$

$$= 828$$

# Egyptian Division

- Suppose we want to divide 25 by 4.
- Start with 1 and the divisor 4.
- Keep doubling both numbers until the right side gets as close as possible to, but not larger than 25.

4	1	
8	2	*
16	4	*

# Egyptian Division

- Subtract the right side numbers from 25 until you can no longer subtract.
- Star the right side numbers that are being subtracted.
  - ✓  $25 - 16 = 9$
  - ✓  $9 - 8 = 1$

4	1	
8	2	*
16	4	*

# Egyptian Division

- What is left is the *remainder*, in this case, the remainder is 1.
- To obtain the answer or *quotient*, add the corresponding left side numbers of the starred positions.  
✓  $4 + 2 = 6$

4	1	
8	2	*
16	4	*

# Egyptian Division

- Thus,  $25 \div 4 = 6 R 1$ .
- The Egyptians would have used unit fractions to write the answer, so for them

$$25 \div 4 = 6\frac{1}{4} = 6 + \frac{1}{4}$$

4	1	
8	2	*
16	4	*

Let's try division the same way:

527 divided by 23

23	1
23+23=46	2 *
46+46=92	4 *
92+92=184	8
184+184=368	16 *
368+368 stop more than 527	

$527 - 368 = 159$  therefore we know that there are at least 16 23s in 527

$159 - 92 = 67$  therefore we know that there are at least 4 more 23s in 527

$67 - 46 = 21$  therefore we know that there are 2 more 23s in 527 with remainder 21.

So, we can say

$$527 = 368 + 92 + 46 + 21 =$$

$$= (16 + 4 + 2)23 + 21$$

Therefore the quotient is 22 and the remainder is 21.

# Try 432 divided by 31.

31	1 *
31+31=62	2
62+62=124	4 *
124+124=248	8 *
248+248=496	stop more than 432

$432 - 248 = 184$  there are at least 8 31s in 432

$184 - 124 = 60$  there are at least 4 more 31s in 432

$60 - 31 = 29$  there is 1 more 31s in 432 with remainder 29

So, we can say

$$\begin{aligned} 432 &= 248 + 124 + 31 + 29 = \\ &= (8 + 4 + 1)31 + 29 \end{aligned}$$

Therefore the quotient is 13 and the remainder is 29.