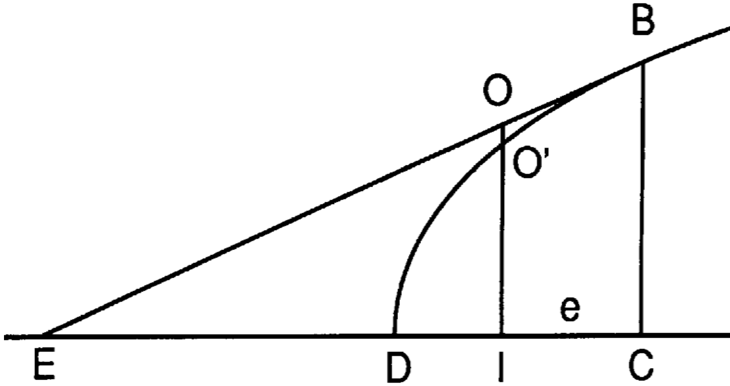
Pierre de Fermat 1601 - 1665

Calculus

René Descartes 1596 - 1650

Fermat



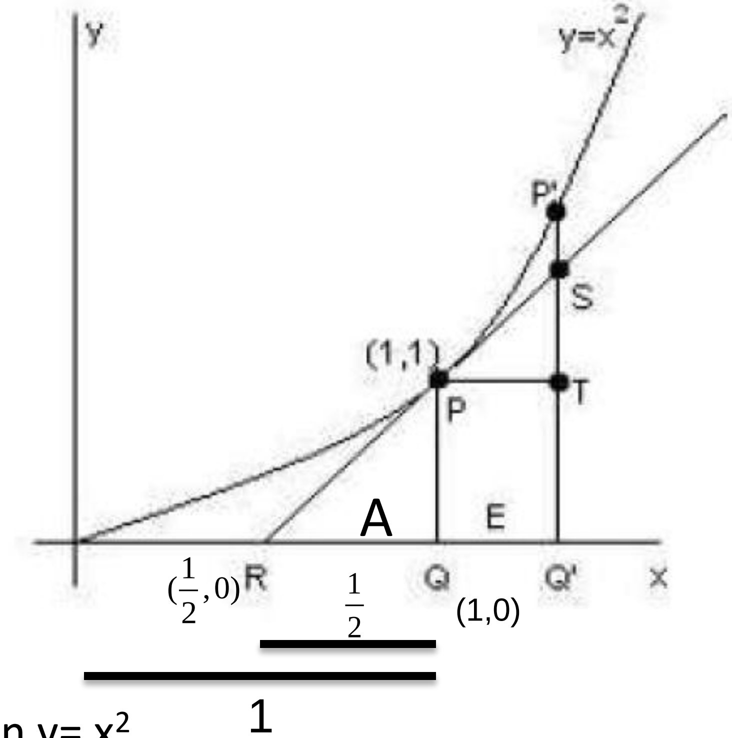
- Fermat used a method called *adequating*, in which he would modify the problem slightly to give a different solution, and then modify again, to find extreme values.
- Given a point *B* on a curve the tangent to have been drawn, intersecting the axis at *E*.
- Let *OI* be drawn parallel to *BC* at a distance *e* from it, intersecting the curve at *O'*.
- Then, on the one hand, the subtangent *EC* is to *BC* as *EI*, i.e. (*EC* - *e*), is to *OI*.

Fermat

$$\frac{a}{a+e} = \frac{QP}{Q'S}$$

We need to find the slope of the tangent line of the function x^2 at the point (1,1)

Fermat compared this to the idea of similar triangles. Therefore we can state the following proportion.



$$\frac{a}{a+e} = \frac{f(1)}{f(1+e)}$$ because we are working with the function $y= x^2$

$$\frac{a}{a+e} = \frac{1^2}{(1+e)^2}$$

$$a+e = a(1+e)^2$$

$$a+e = a+2ae+ae^2$$

$$e = 2ae+ae^2$$

Divide both sides by e

$$1 = 2a + ae$$ Now, e is very small

Let e be so small that it is zero. (they did not have limits)

Therefore,

the length of $a = \frac{1}{2}$

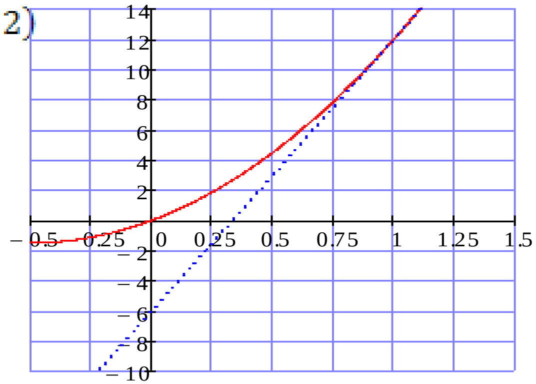
Since Q is the point (1,0) and a=1/2 then R is 1-(1/2)= ½ then R is (1/2,0)

Therefore we just need to write an equation of a line that passes through the point s (1/2,0) and (1,1) .

y=2x – 1

Let's try another one.

$$y = 6x^2 + 6x \ \ at\ the\ point \quad (1{,}12)$$



$$\frac{a}{a+e} = \frac{f(1)}{f(1+e)}$$

$$\frac{a}{a+e} = \frac{6(1)^2 + 6(1)}{6(1+e)^2 + 6(1+e)}$$

$$\frac{a}{a+e} = \frac{12}{6(1+e)^2 + 6(1+e)}$$

$$12a + 12e = 6a(1+e)^2 + 6a(1+e)$$

$$12a + 12e = 12a + 18ae + 6ae^2$$

$$-12a \qquad -12a$$

$$\frac{12e}{12e} = \frac{18ae}{12e} + \frac{6ae^2}{12e}$$

$1 = \frac{3}{2}a + \frac{1}{2}ae$ let e be so small it equals 0

$$1 = \frac{3}{2}a$$

$$\left(\frac{2}{3}\right)1 = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)a$$

$$a = \frac{2}{3}$$

My point x – a gives us: $\frac{1}{3}$

The two points we will use are now:
(1, 12) and (1/3, 0) therefore the line is:

Slope = $\frac{12-0}{1-\frac{1}{3}} = 18$

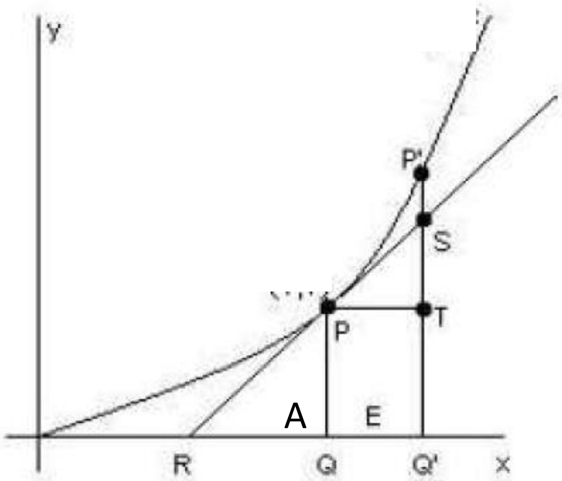
$$y = mx + b$$

$$12 = 18x + b$$

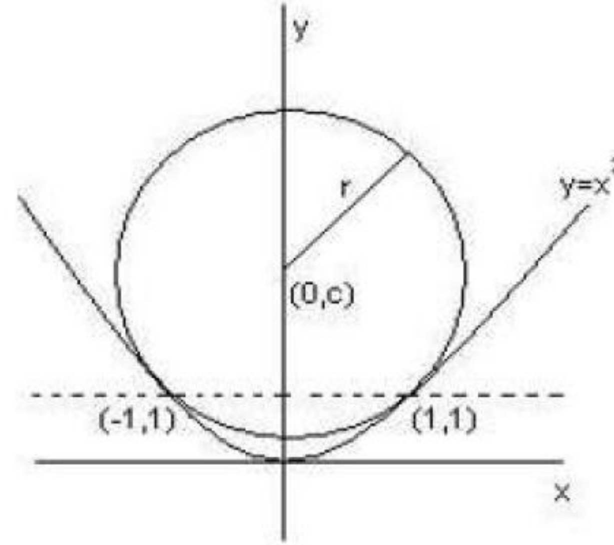
$$12 = 18(1) + b$$

$$-6 = b$$

$$y = 18x - 6$$



DeCartes



Find the center of s circle whose center is on the y-axis, such that it is tangent to the parabola .

We need the equation of a circle.

$$(x-x_1)^2+(y-y_1)^2=r^2$$

We need a center, and a radius.

Center (0,c) and radius is ?

Length between (0,c) and (1,1).

$$r=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$r=\sqrt{1+(c-1)^2}$$

$$r^2=1+(c-1)^2$$

Now let's substitute into the formula of a circle.

$$(x-x_1)^2+(y-y_1)^2=r^2$$

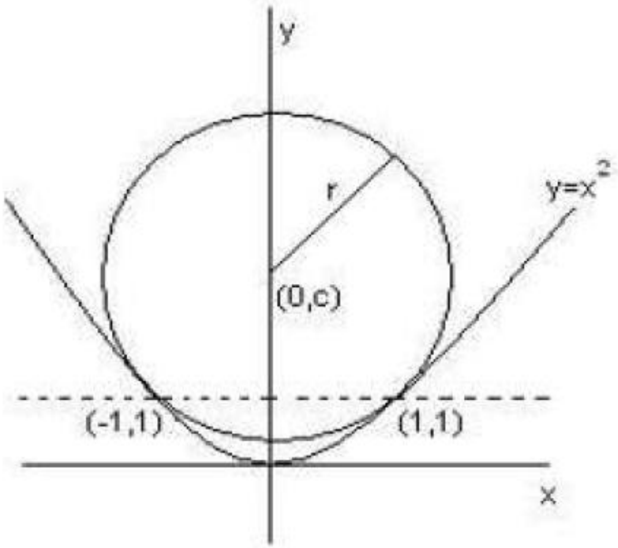
$$(x-0)^2+(y-c)^2=(c-1)^2+1$$

Since we know the point is on the circle, $y=x^2$
Substitute in for x and simplify.

$$y+y^2-2yc+c^2=c^2-2c+2$$

$$y^2-2yc+y+2c-2=0$$

DeCartes



$$y^2 + (-2c+1)y + 2c - 2 = 0$$

Because it is a tangent, we want their to be only one solution.
How do we find c such that there is only one solution?
The discriminate must equal 0.

$A = 1 \qquad B = -2c+1 \qquad C = 2c-2$

$$B^2 - 4AC$$

$$(-2c+1)^2 - 4(1)(2c-2) = 0$$

$$4c^2 - 12c + 9 = 0$$

$(2c-3)^2 = 0 \quad c = 1.5$

$$center(0,1.5)$$

Now let’s substitute into the formula of a circle.

$$(x-x_1)^2 + (y-y_1)^2 = r^2$$

$$(x-0)^2 + (y-c)^2 = (c-1)^2 + 1$$

Since we know the point is on the circle, $y = x^2$
Substitute in for x and simplify.

$$y + y^2 - 2yc + c^2 = c^2 - 2c + 2$$

$$y^2 - 2yc + y + 2c - 2 = 0$$