Pascal's Triangle

1628-1662

Pascal’s Wager

Even though the existence of God cannot be determined through reason, a person should wager as though God exists, because living life accordingly has everything to gain, and nothing to lose. Although Pascal formulated his suggestion on the Judeo-Christian God, this wager can apply to the god of all other religions as well.

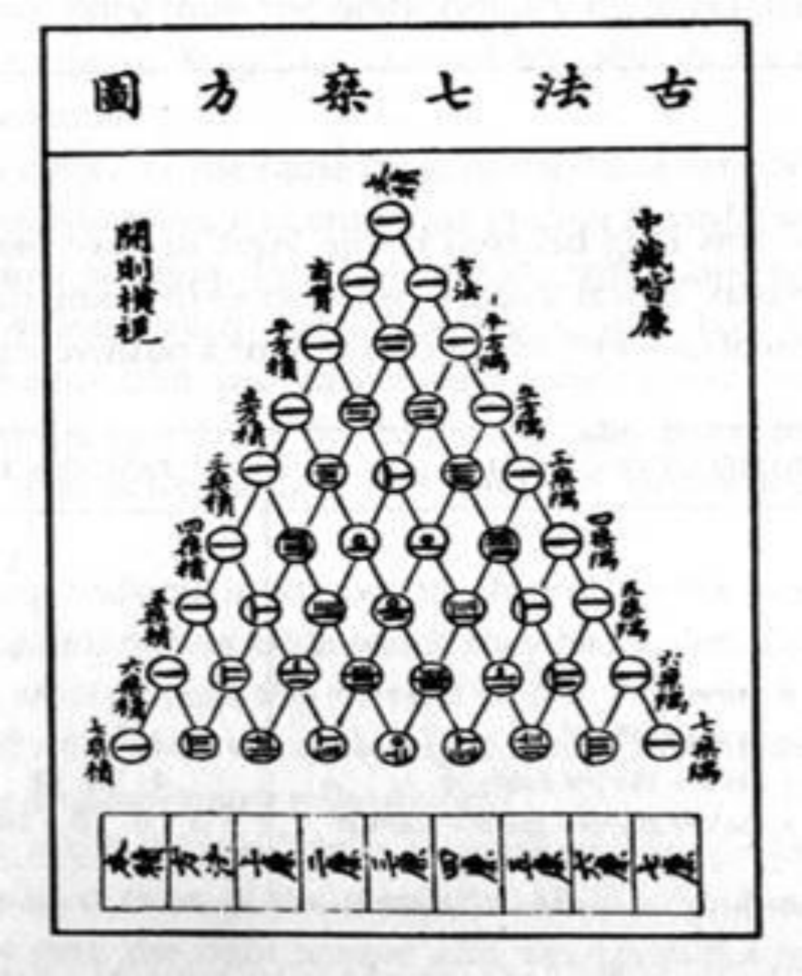
		Existence of God	
		No	Yes
Personal Conduct	Righteous	Oh Well	Whew
	Sinful	Fun (short-term)	Damn (eternal)

His original triangle was published in 1654, and it looked like

1	1	1	1	1	1	1	.	.	.
1	2	3	4	5	6	.	.	.	.
1	3	6	10	15	.	.	.	.	.
1	4	10	20	.	.	.	.	.	.
1	5	15	.	.	.	.	.	.	.
1	6	.	.	.	.	.	.	.	.
1	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.

Chu Shih Chieh

- The Chinese mathematician Chu Shih Chieh depicted the triangle and indicated its use in providing coefficients for the binomial expansion of in his 1303 treatise *The Precious Mirror of the Four Elements*.



Pascal?

- A similar arrangement of binomial coefficients was known by Chinese around 1000 C.E
- The Persian mathematician Omar Khayyam who lived in the 11th or 12th century included such triangle in his writings.
- Pascal gets credit, but the arrangement was know by earlier Eastern mathematicians.
- Pascal was the first to make a systematic study of the patterns involved.
- He listed 19 properties of binomial coefficients that he discovered from the triangle.

Connections

There is connection between Pascal's triangle, binomial coefficients, and Combinations in Probability.

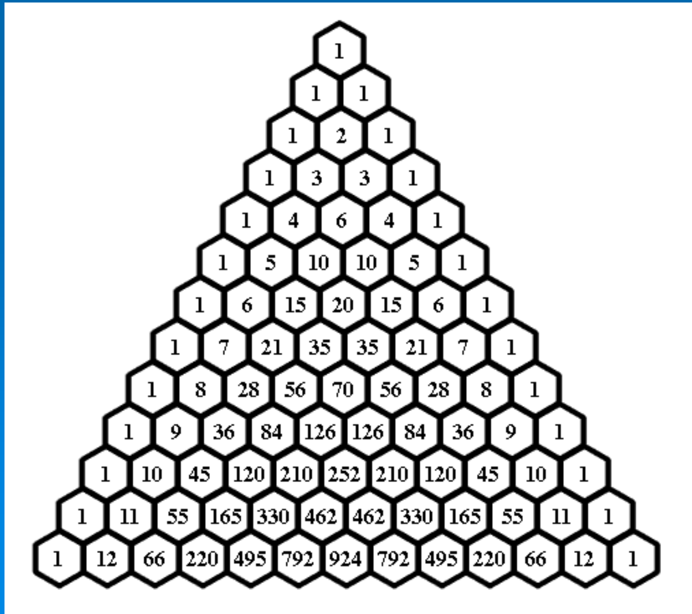
The row 1, 6, 15, 20, 15, 6, 1 is also

$$\binom{6}{0},\binom{6}{1},\binom{6}{2},\binom{6}{3},\binom{6}{4},\binom{6}{5},\binom{6}{6}$$

$$(x+y)^6 = 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$$

$$(a+b)^8 = 1a^8b^0 + 8a^7b^1 + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8a^1b^7 + 1a^0b^8$$

Suppose you are planning to have lunch. You have three items, a hamburger, an avocado, and some bananas that you can choose for lunch. You can choose to have one item for lunch, 2 items for lunch or all three. Or you can choose not to eat lunch. How many different possible lunches are there?

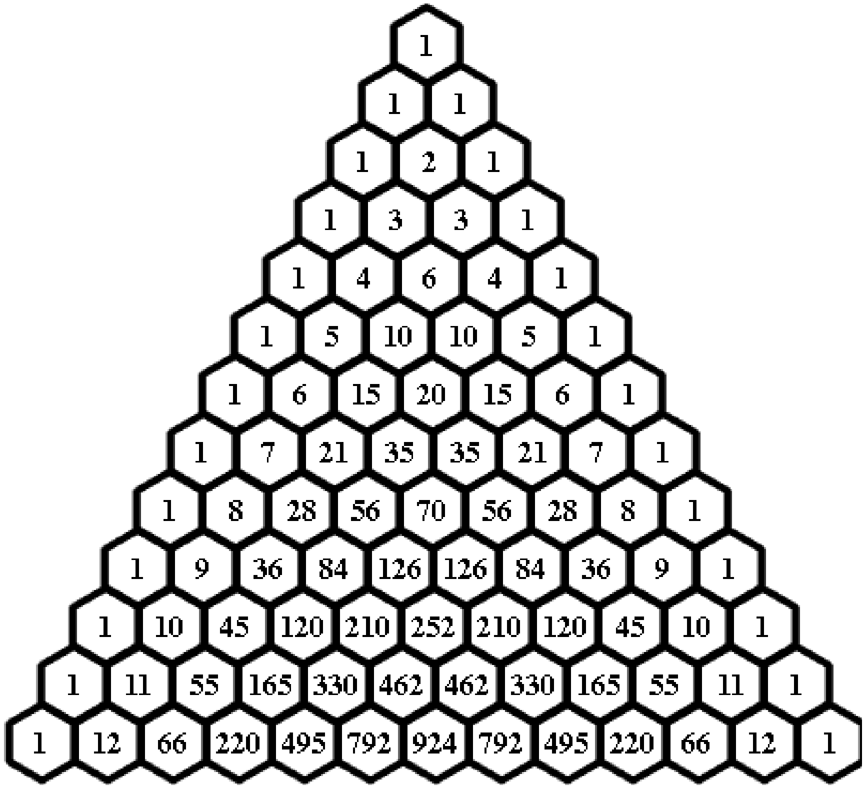


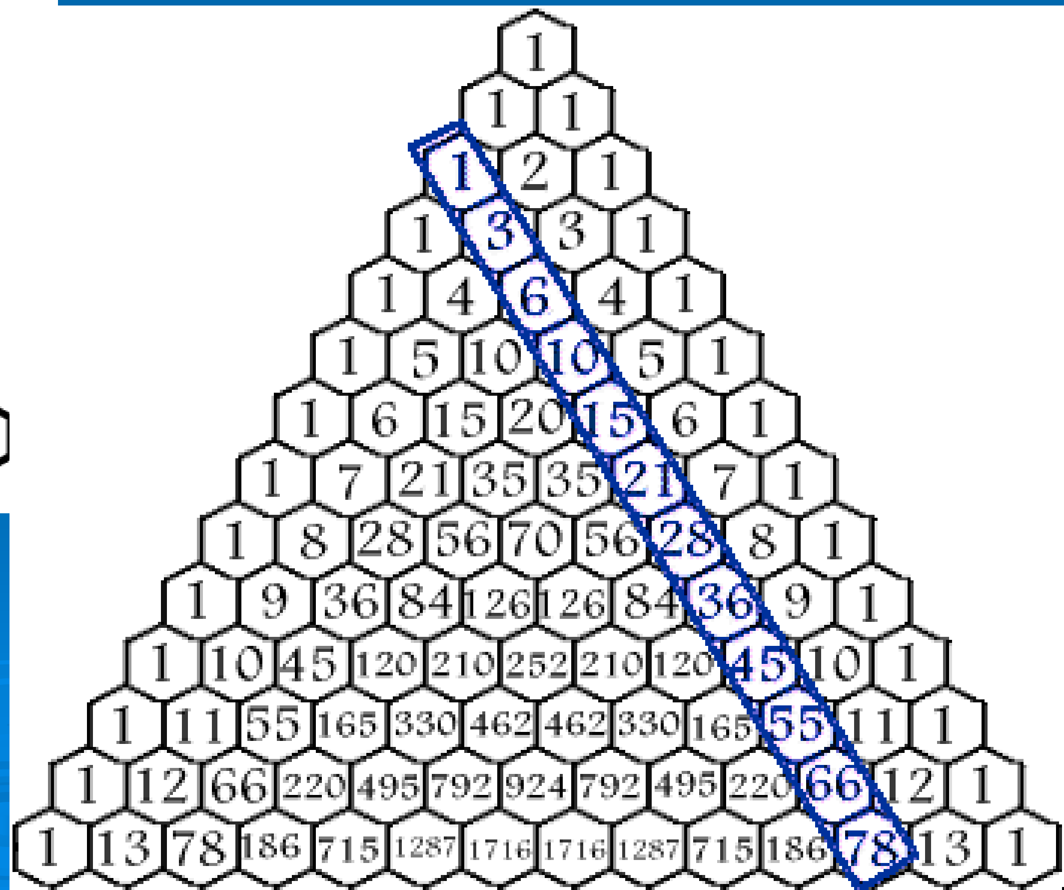
Number of Items in Lunch	Possible Lunches	Number of Possible Lunches
3		1
2		3
1		3
0		1

Consider the same problem, but with four items to choose from. Suppose in addition to the hamburger, avocado and bananas, there are also some cherries:

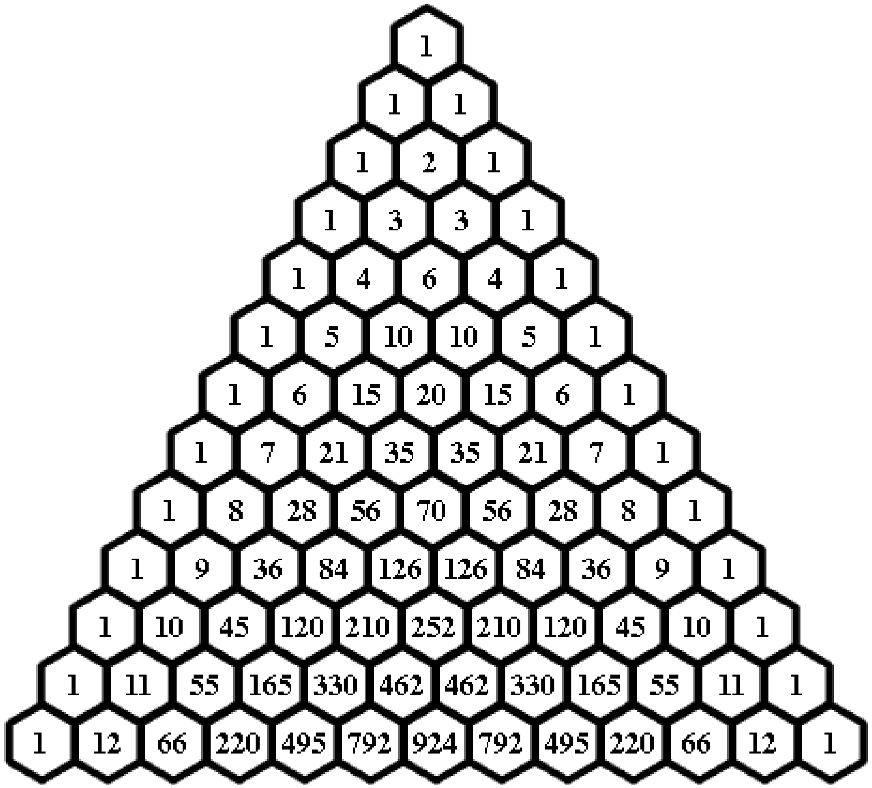
Number of Items in Lunch	Possible Lunches	Number of Posible Lunches
4		1
3		4
2		6
1		4
0		1

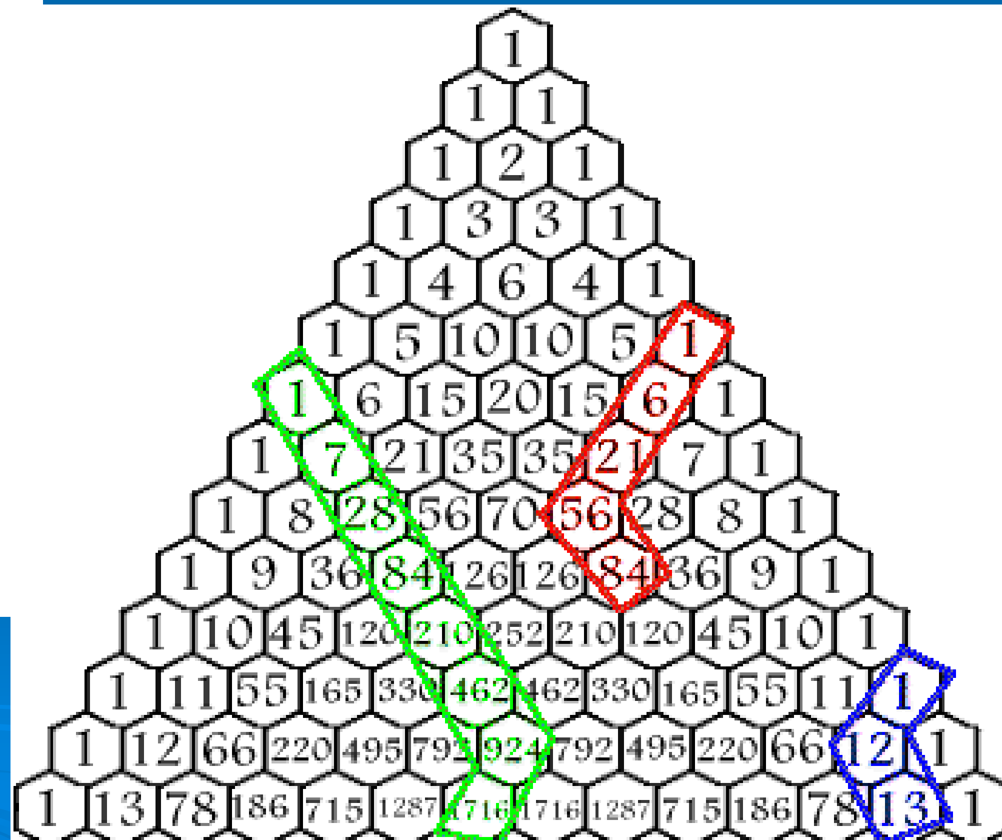
Triangular Numbers



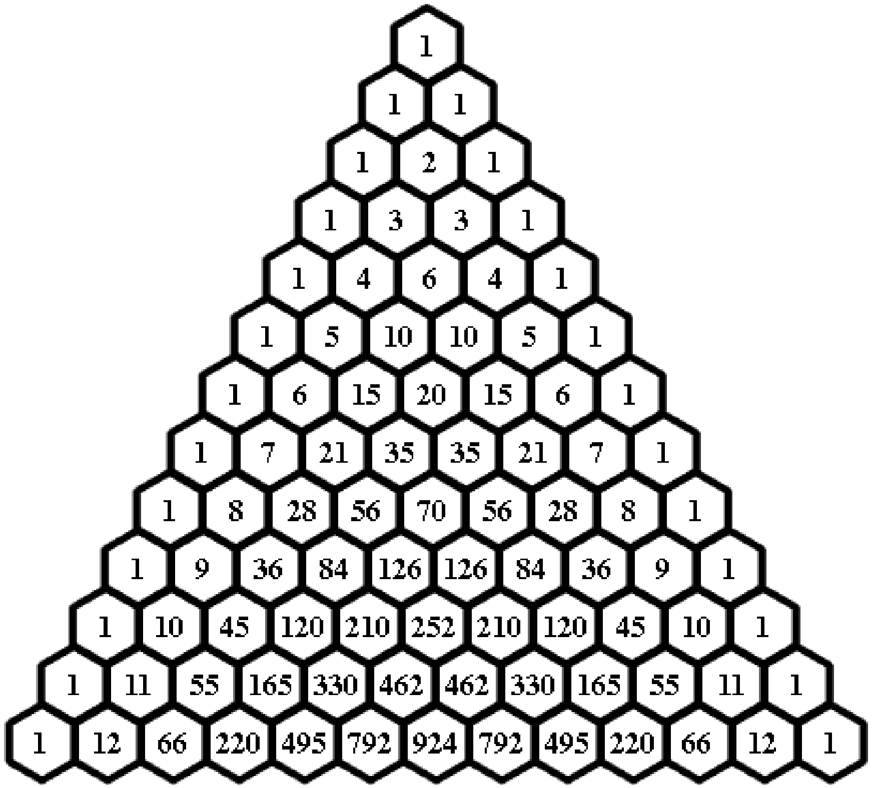


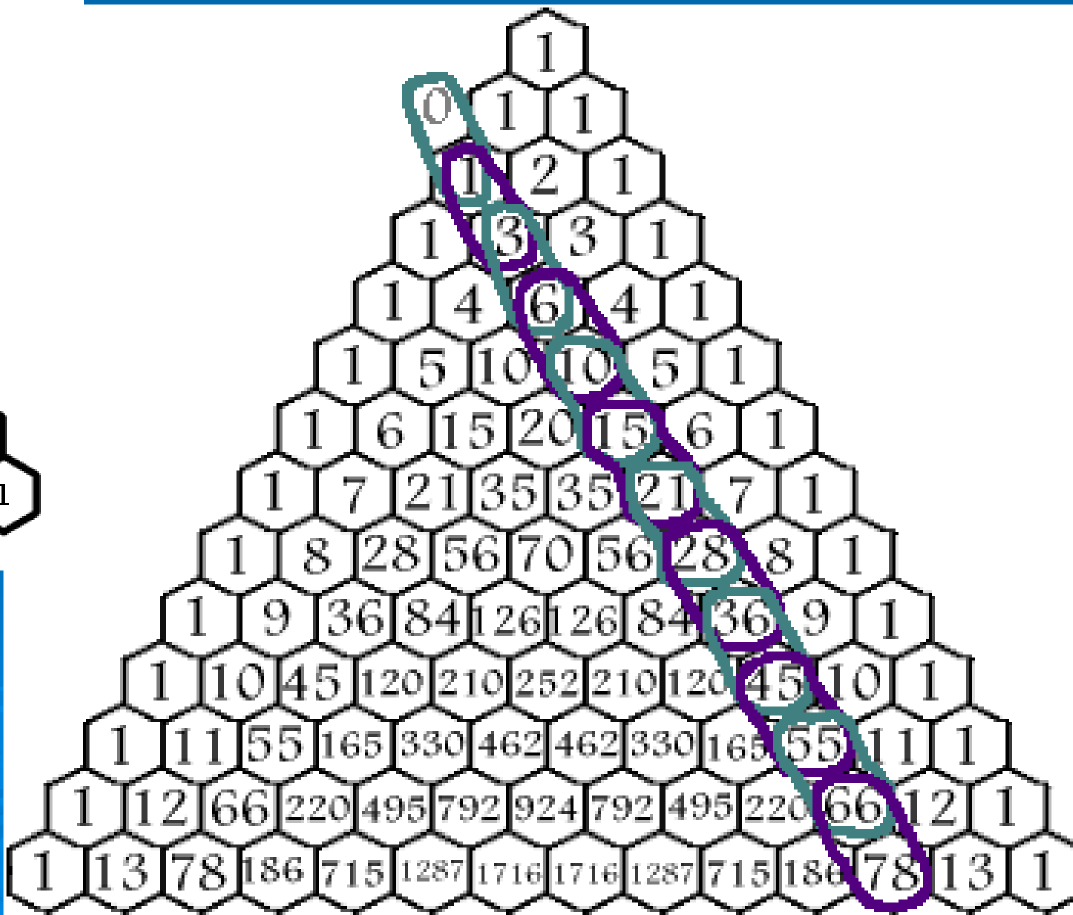
Hockey Stick





Square Numbers

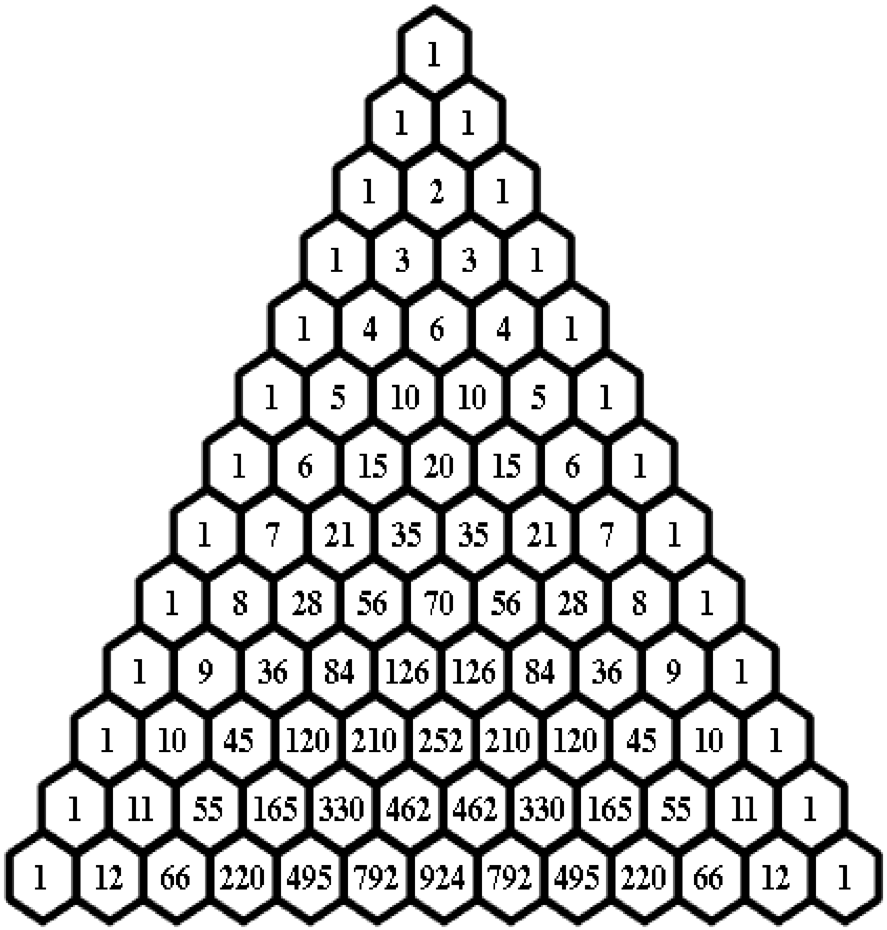




•If the first element in a row is a prime number (remember, the 0th element of every row is 1), all the numbers in that row (excluding the 1's) are divisible by it. For example, in row 7 (1 7 21 35 35 21 7 1) 7, 21, and 35 are all divisible by 7.

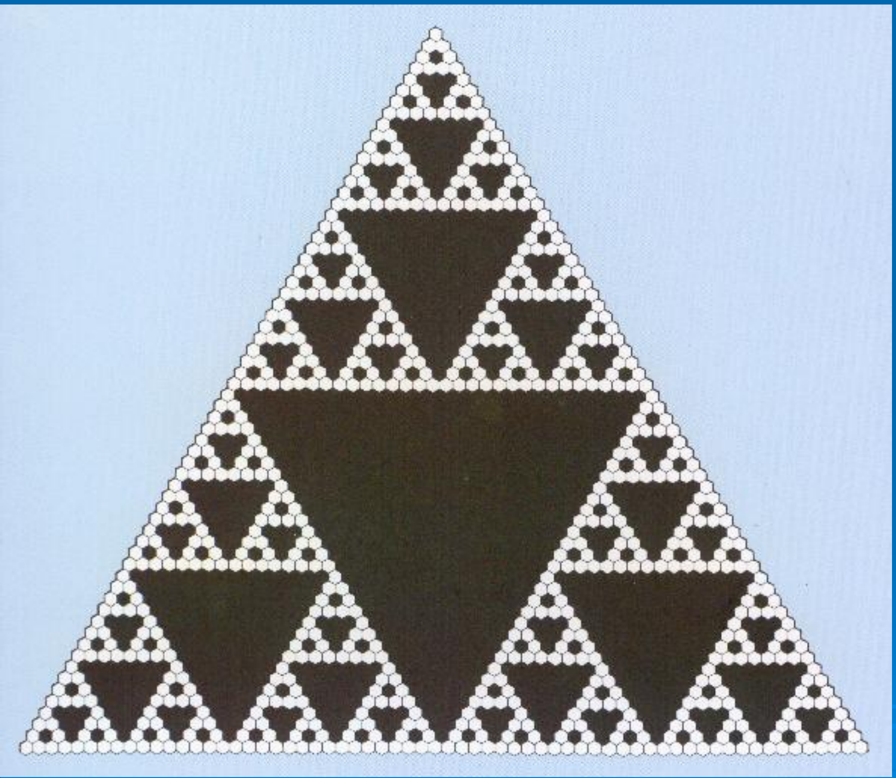
•The sum of the numbers in any row is equal to 2 to the nth power, when n is the number of the row. For example:

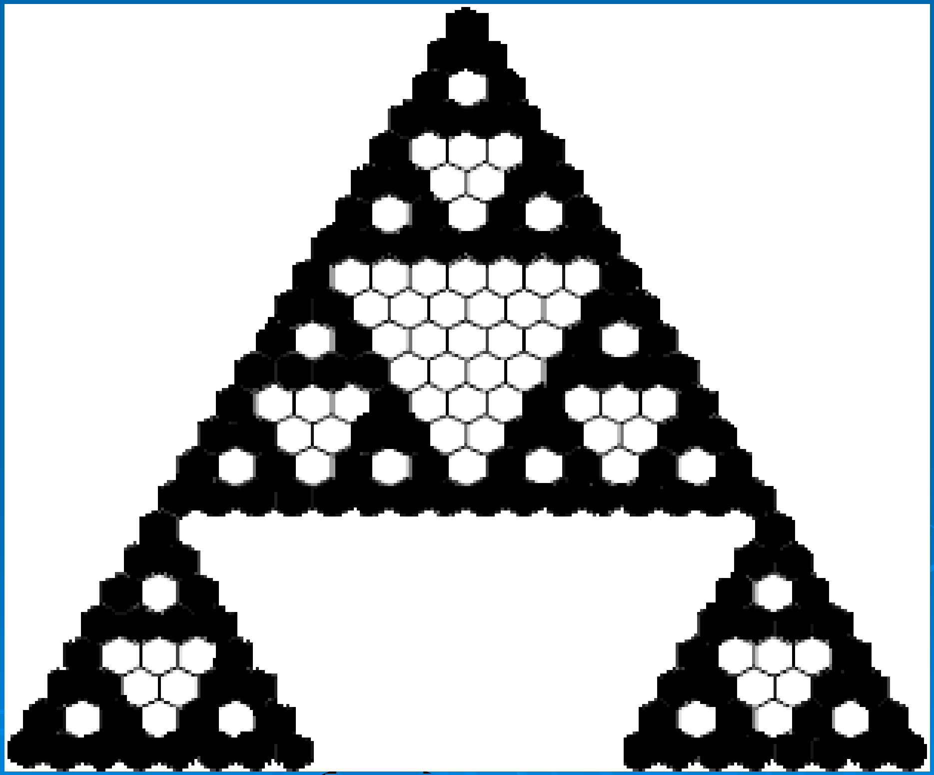
= 1
= 1+1 = 2
=1+2+1 = 4
= 1+3+3+1 = 8
= 1+4+6+4+1 = 16



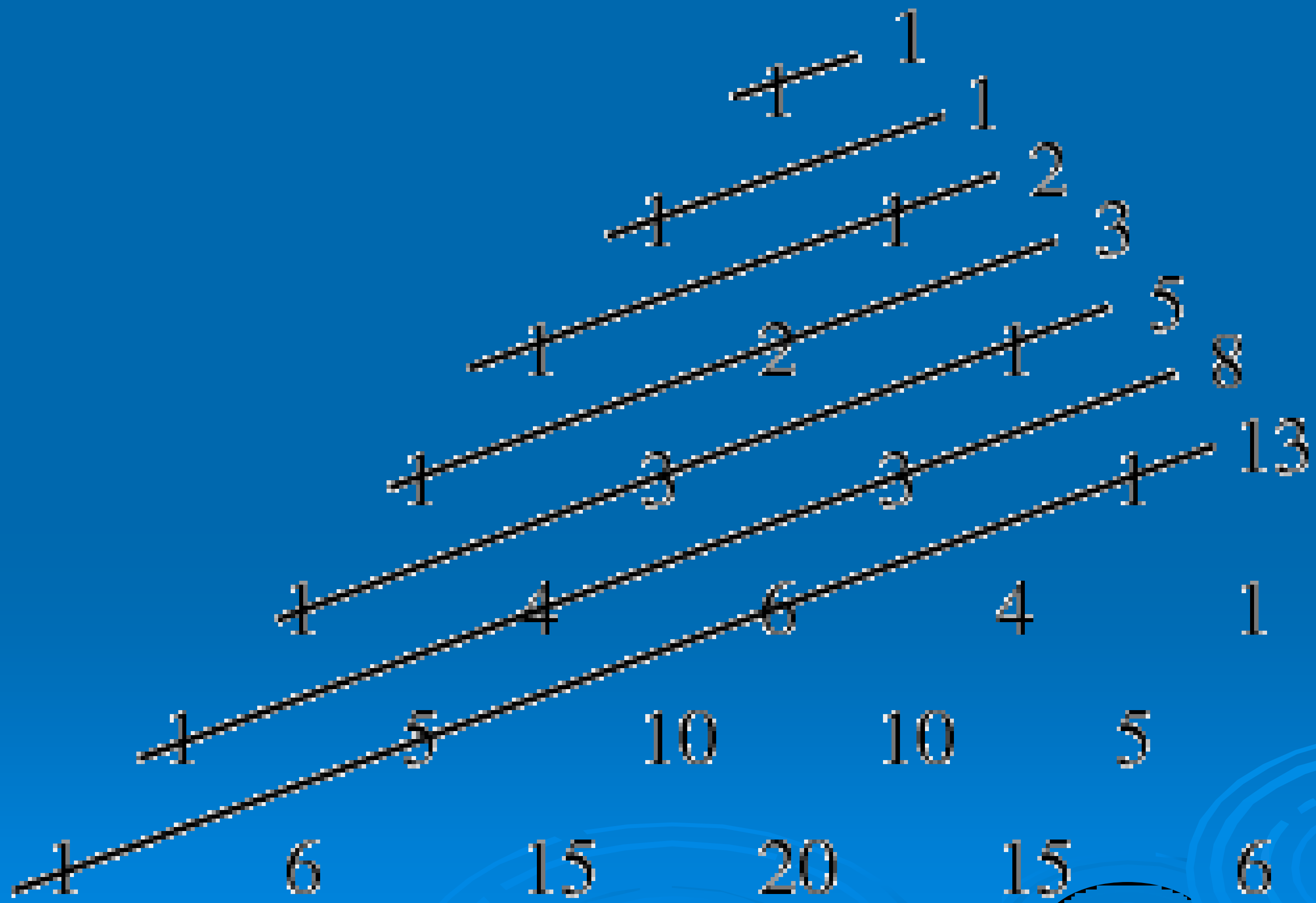
When all the odd numbers (numbers not divisible by 2) in **Pascal's Triangle** are filled in (black) and the rest (the evens) are left blank (white), the recursive

Sierpinski **Triangle** fractal is revealed





Fibonacci Numbers



Pascal Petals

- Notice that the gray cell is surrounded by 6 other cells. These six cells make up the petals on Pascal's flower.
- Starting with the petal above and to the left of the gray center, alternating petals are colored yellow and numbered 5, 20, and 21.
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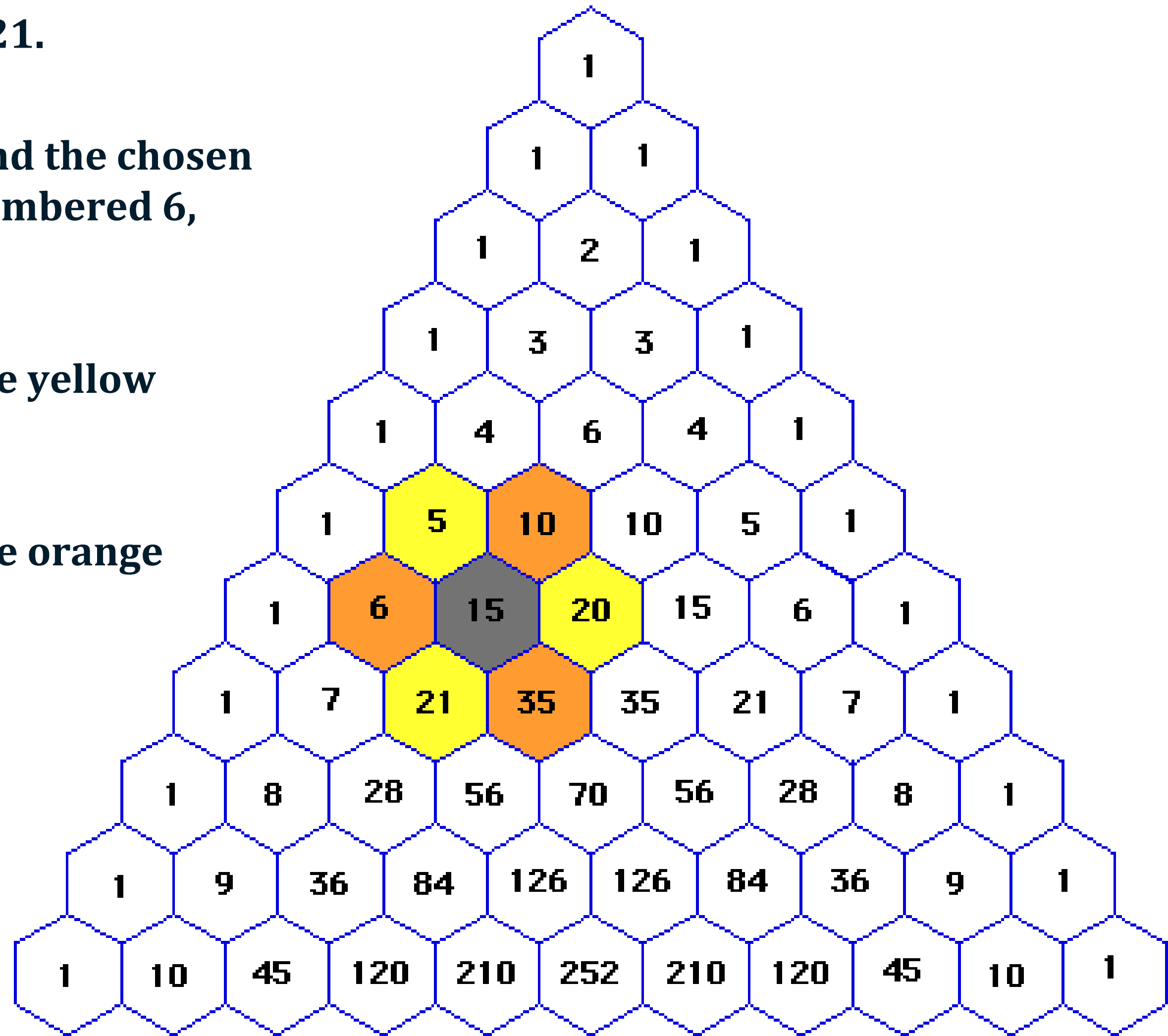
- The three remaining petals around the chosen center are colored orange and numbered 6, 10, and 35.
- The product of the numbers in the yellow petals is 5 x 20 x 21 = 2100.
- The product of the numbers in the orange petals is 6 x 10 x 35 = 2100.



- http://home.covad.net/~bfjacobs/Student_Info/Math_Fun/Patterns/Pascal's_Triangle/usingpascal's.html