Algebra Tiles

Arabic/Islamic Mathematics

- Algebra gave mathematics a whole new development path so much broader in concept to that which had existed before, and provided a vehicle for future development of the subject.
- Another important aspect of the introduction of algebraic ideas was that it allowed mathematics to be applied to itself in a way which had not happened before.
- All of this was done despite not using symbols.

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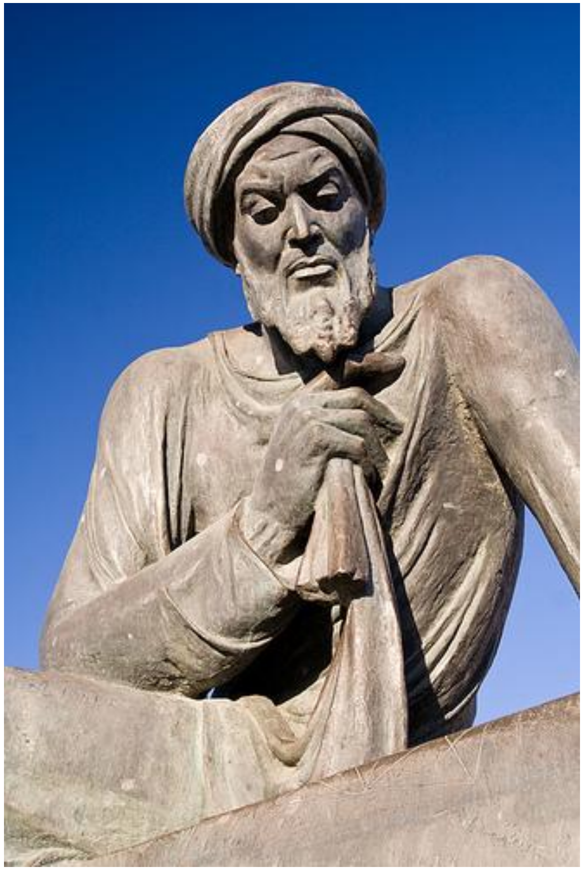
- All of this was done despite not using symbols.

Muhammad ibn-Mūsā Al Khwārizmī (ca. 780-850 AD)

Statue of Muhammad ibn Mūsā al-Khwārizmī, sitting in front of Khiva, north west of Uzbekistan.



- Sometimes called the "*Father of Algebra*".
- His most important work entitled *Al-kitāb al-muhtasar fī hisāb al-jabr wa-l-muqābala* was written around 825.

Muhammad ibn-Mūsā Al Khwārizmī (ca. 780-850 AD)

- The word algebra we use today comes from *al-jabr* in the title.
- The translated title is "*The Condensed Book on the Calculation of al-Jabr and al-Muqabala*."

Muhammad ibn-Mūsā Al Khwārizmī (ca. 780-850 AD)

- The word *al-jabr* means “restoring”, “reunion”, or “completion” which is the process of transferring negative terms from one side of an equation to the other.
- The word *al-muqabala* means “reduction” or “balancing” which is the process of combining like terms on the same side into a single term or the cancellation of like terms on opposite sides of an equation.

Muhammad ibn-Mūsā Al Khwārizmī (ca. 780-850 AD)

- He classified the solution of quadratic equations and gave geometric proofs for completing the square.
- This early Arabic algebra was still at the primitive rhetorical stage – No symbols were used and no negative or zero coefficients were allowed.
- He divided quadratic equations into three cases

  $x^2 + ax = b$, $x^2 + b = ax$, and $x^2 = ax + b$ with only positive coefficients.

Muhammad ibn-Mūsā Al Khwārizmī (ca. 780-850 AD)

- Solve $x^2 + 10x = 39$.
- Construct a square having sides of length x to represent x^2.
- Then add $10x$ to the x^2, by dividing it into 4 parts each representing $10x/4$.
- Add the 4 little $10/4 \times 10/4$ squares, to make a larger $x + 10/2$ side square.

Completing the Square

- By computing the area of the square in two ways and equating the results we get the top equation at the right.
- Substituting the original equation and using the fact that $a^2 = b$ implies $a = \sqrt{b}$.

$$\left(x+\frac{10}{2}\right)^2 = \left(x^2+10x\right)+4\left(\frac{10}{4}\right)^2$$

$$= 39+\left(\frac{10}{2}\right)^2$$

$$= 39+25 = 64$$

$$\Rightarrow x+\frac{10}{2} = 8$$

$$\Rightarrow x = 3$$

Algebra Tiles

- Manipulatives used to enhance student understanding of subject traditionally taught at symbolic level.
- Provide access to symbol manipulation for students with weak number sense.
- Provide geometric interpretation of symbol manipulation.

Algebra Tiles

- Support cooperative learning, improve discourse in classroom by giving students objects to think with and talk about.
- **When I listen, I hear.**
- **When I see, I remember.**
- **But when I do, I understand.**

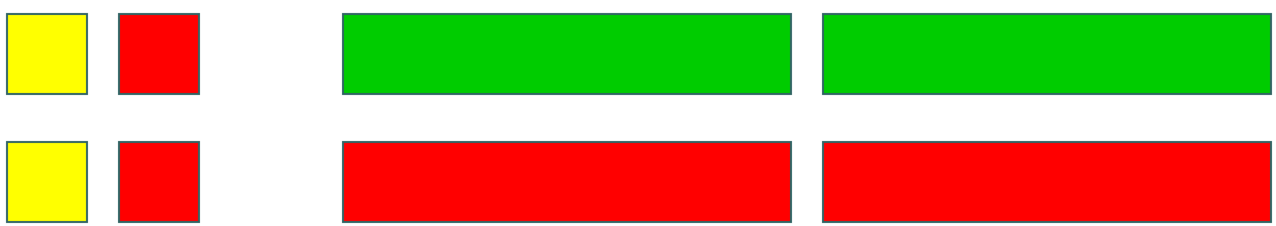
Algebra Tiles

- Algebra tiles can be used to model operations involving integers.
- Let the small yellow square represent +1 and the small red square (the flip-side) represent -1.

- The yellow and red squares are additive inverses of each other.

Zero Pairs

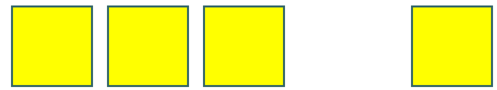
- Called zero pairs because they are additive inverses of each other.
- When put together, they cancel each other out to model zero.

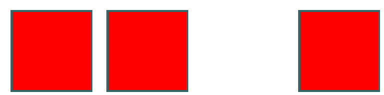


Addition of Integers

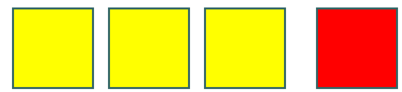
- Addition can be viewed as "combining".
- Combining involves the forming and removing of all **zero pairs**.
- For each of the given examples, use algebra tiles to model the addition.
- Draw pictorial diagrams which show the modeling.

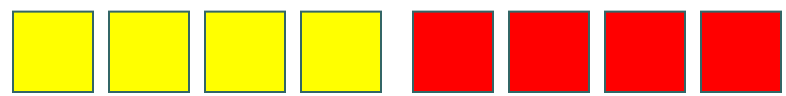
Addition of Integers

(+3) + (+1) = 

(-2) + (-1) = 

Addition of Integers

(+3) + (-1) = 

(+4) + (-4) = 

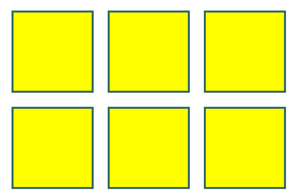
- After students have seen many examples of addition, have them formulate rules.

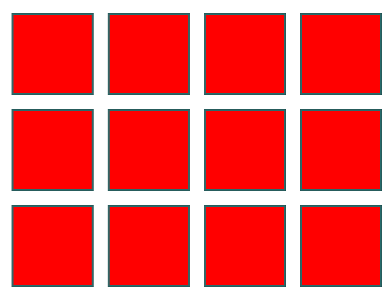
Multiplication of Integers

- Integer multiplication builds on whole number multiplication.
- Use concept that the multiplier serves as the “counter” of sets needed.
- For the given examples, use the algebra tiles to model the multiplication. Identify the multiplier or counter.
- Draw pictorial diagrams which model the multiplication process.

Multiplication of Integers

- The counter indicates how many rows to make. It has this meaning if it is positive.

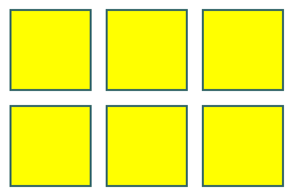
(+2)(+3) = 

(+3)(-4) = 

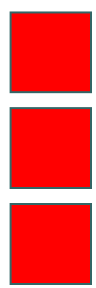
Multiplication of Integers

- If the counter is negative it will mean "take the opposite of." (flip-over)

(-2)(+3)



(-3)(-1)



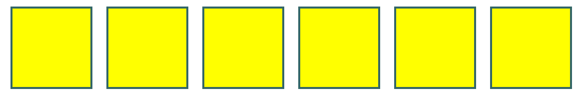
Multiplication of Integers

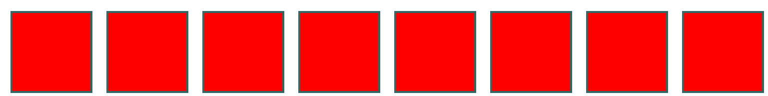
- After students have seen many examples, have them formulate rules for integer multiplication.
- Have students practice applying rules abstractly with larger integers.

Division of Integers

- Like multiplication, division relies on the concept of a counter.
- Divisor serves as counter since it indicates the number of rows to create.
- For the given examples, use algebra tiles to model the division. Identify the divisor or counter. Draw pictorial diagrams which model the process.

Division of Integers

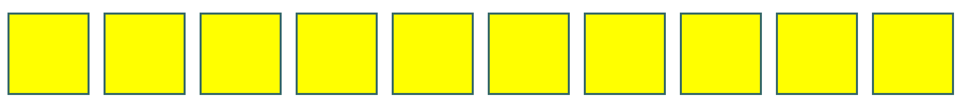
(+6)/(+2) = 

(-8)/(+2) = 

Division of Integers

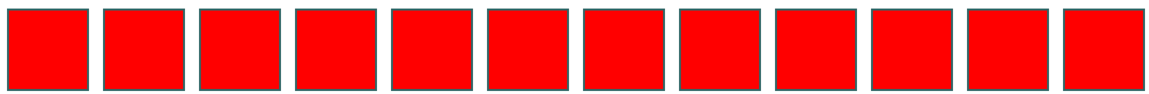
- A negative divisor will mean "take the opposite of." (flip-over)

(+10)/(-2) =



Division of Integers

(-12)/(-3) =



- After students have seen many examples, have them formulate rules.

Solving Equations

- Algebra tiles can be used to explain and justify the equation solving process. The development of the equation solving model is based on two ideas.
- Variables can be isolated by using zero pairs.
- Equations are unchanged if equivalent amounts are added to each side of the equation.

Solving Equations

- Use the green rectangle as X and the red rectangle (flip-side) as –X (the opposite of X).

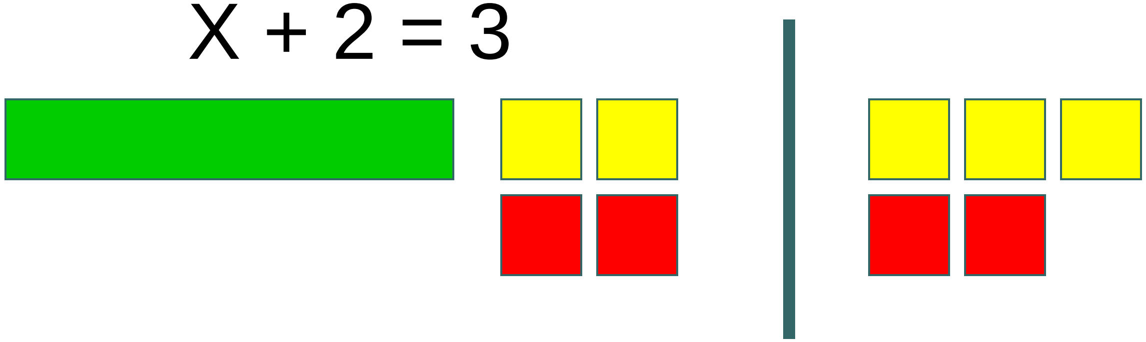
$X + 2 = 3$

$2X - 4 = 8$

$2X + 3 = X - 5$

Solving Equations

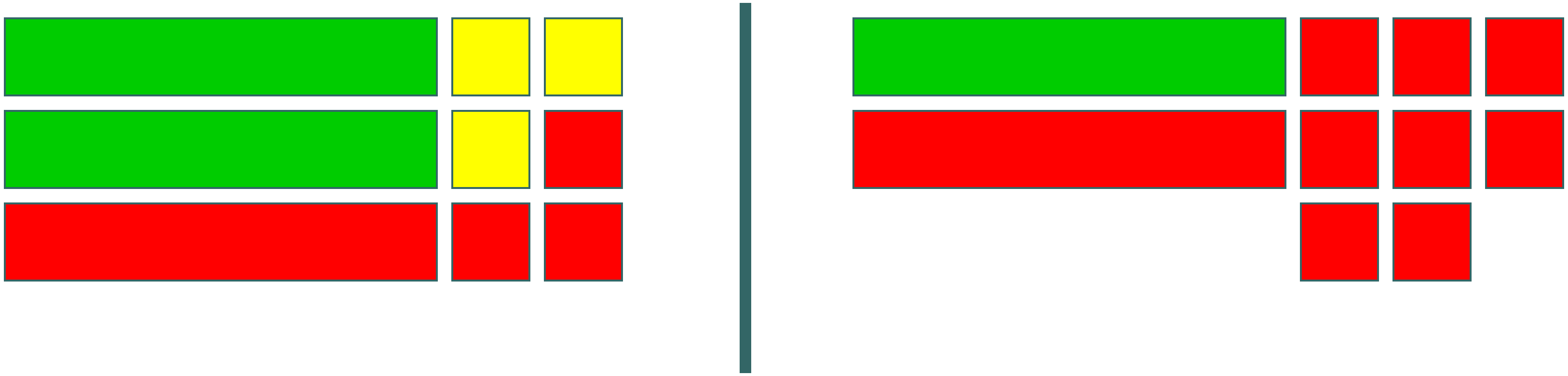
$X + 2 = 3$



$2X - 4 = 8$

Solving Equations

2X + 3 = X – 5



Distributive Property

- Use the same concept that was applied with multiplication of integers, think of the first factor as the counter.
- The same rules apply.

$3(X+2)$

- Three is the counter, so we need three rows of $(X+2)$

Distributive Property

3(X + 2)

3(X – 4)

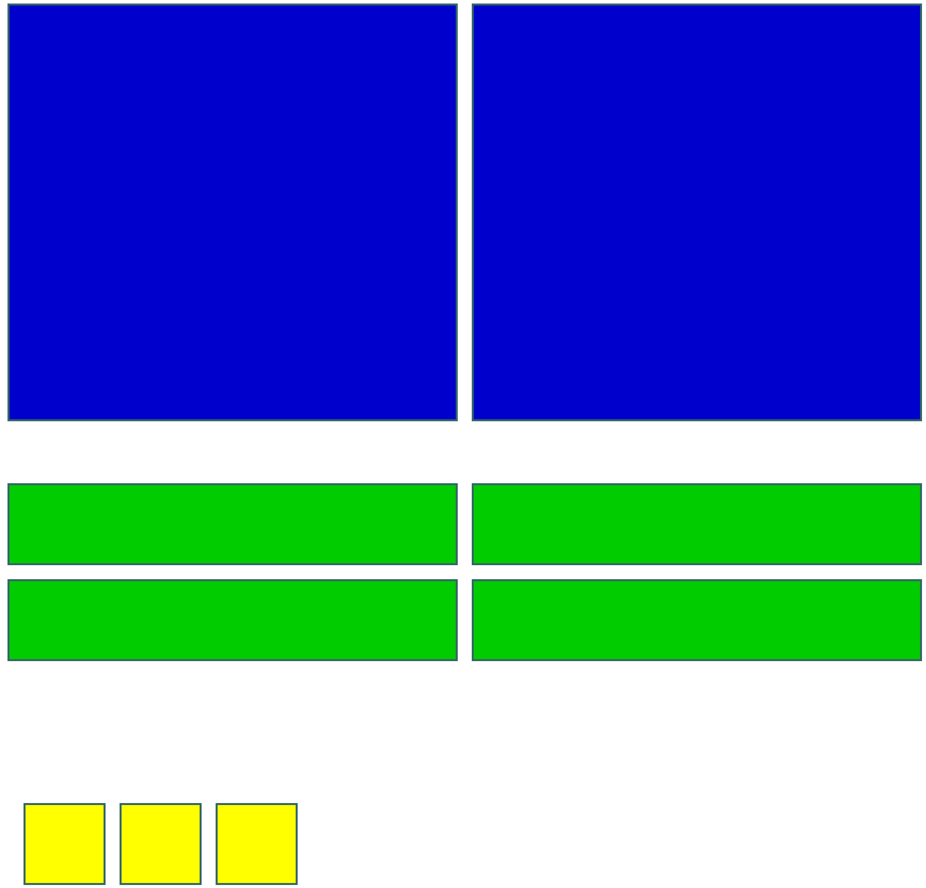
-2(X + 2)

-3(X – 2)

Modeling Polynomials

- Algebra tiles can be used to model expressions.
- Aid in the simplification of expressions.
- Add, subtract, multiply, divide, or factor polynomials.

Modeling Polynomials



$2x^2$

$4x$

3

Modeling Polynomials

$3x^2 + 5y^2$

$-2xy$

$-3x^2 - 4xy$

- Textbooks do not always use x and y. Use other variables in the same format. Model these expressions.

$-a^2 + 2ab$

$5p^2 - 3pq + q^2$

More Polynomials

- Would not present previous material and this information on the same day.
- Let the blue square represent x^2 and the large red square (flip-side) be $–x^2$.
- Let the green rectangle represent x and the red rectangle (flip-side) represent –x.
- Let yellow square represent 1 and the small red square (flip-side) represent –1.

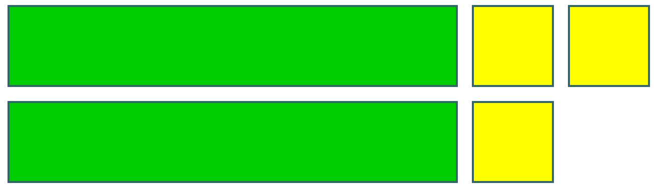
More Polynomials

- Represent each of the given expressions with algebra tiles.
- Draw a pictorial diagram of the process.
- Write the symbolic expression.

x + 4

More Polynomials

$2x + 3$



$4x - 2$

More Polynomials

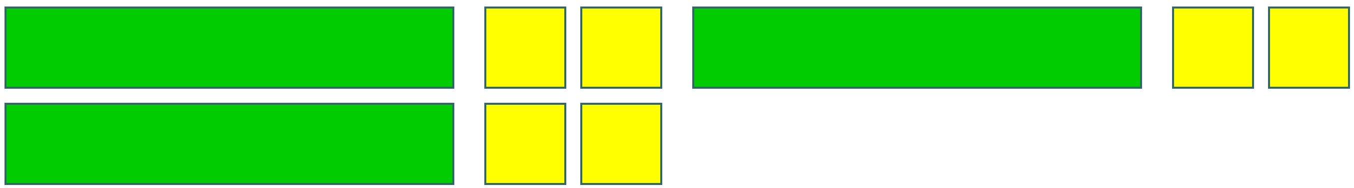
- Use algebra tiles to simplify each of the given expressions. Combine like terms. Look for zero pairs. Draw a diagram to represent the process.
- Write the symbolic expression that represents each step.

$2x + 4 + x + 2$

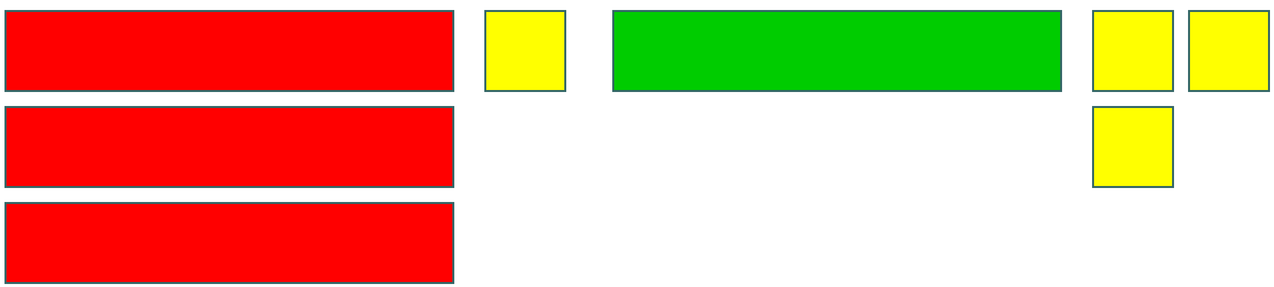
$-3x + 1 + x + 3$

More Polynomials

2x + 4 + x + 2



-3x + 1 + x + 3



More Polynomials

$3x + 1 – 2x + 4$

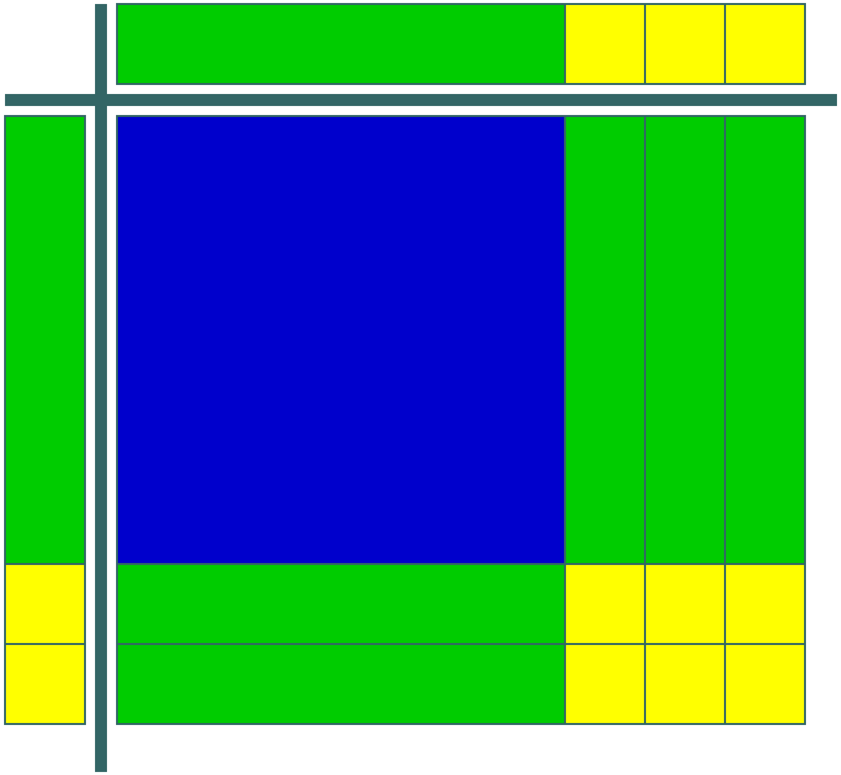
- This process can be used with problems containing x^2.

$(2x^2 + 5x – 3) + (-x^2 + 2x + 5)$

$(2x^2 – 2x + 3) – (3x^2 + 3x – 2)$

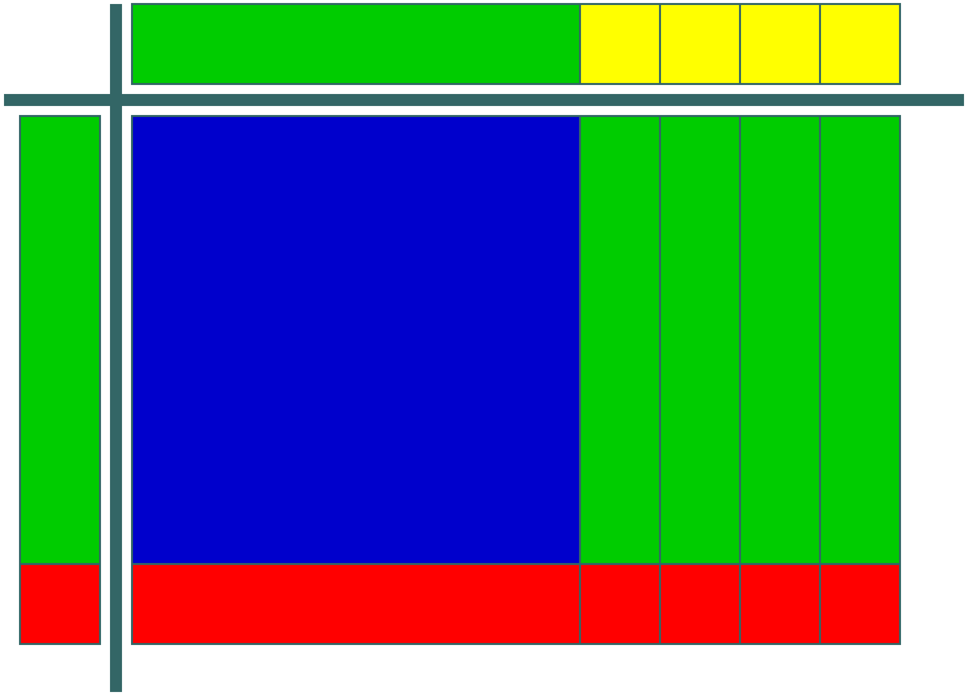
Multiplying Polynomials

(x + 2)(x + 3)



Multiplying Polynomials

$(x - 1)(x + 4)$



Multiplying Polynomials

$(x + 2)(x - 3)$

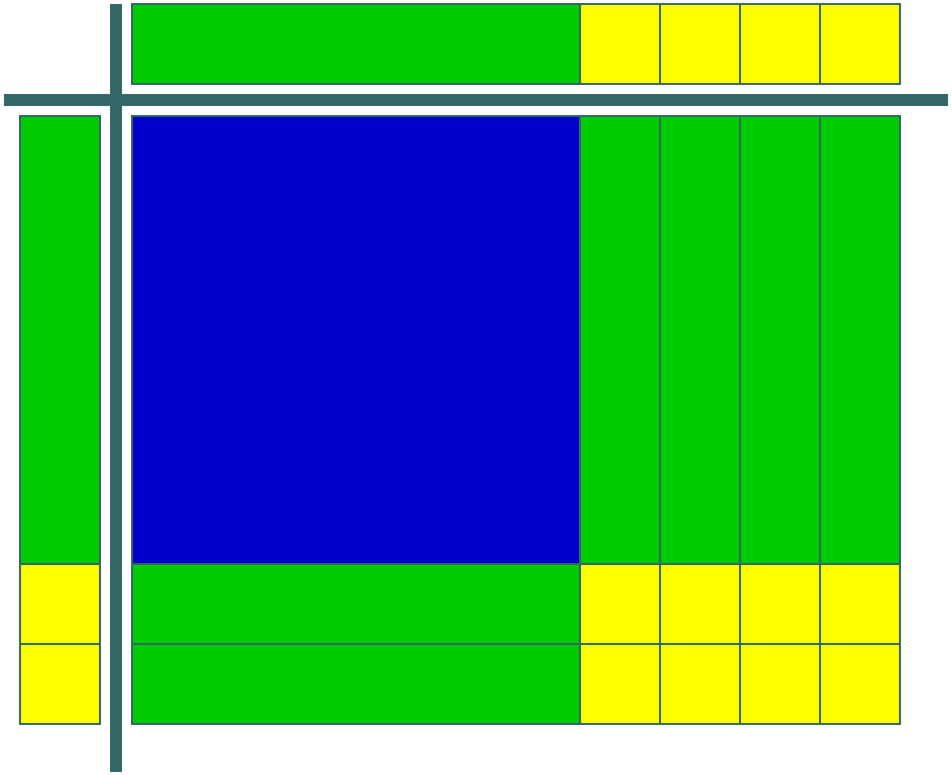
$(x - 2)(x - 3)$

Factoring Polynomials

- Algebra tiles can be used to factor polynomials. Use tiles and the frame to represent the problem.
- Use the tiles to fill in the array so as to form a rectangle inside the frame.
- Be prepared to use zero pairs to fill in the array.
- Draw a picture.

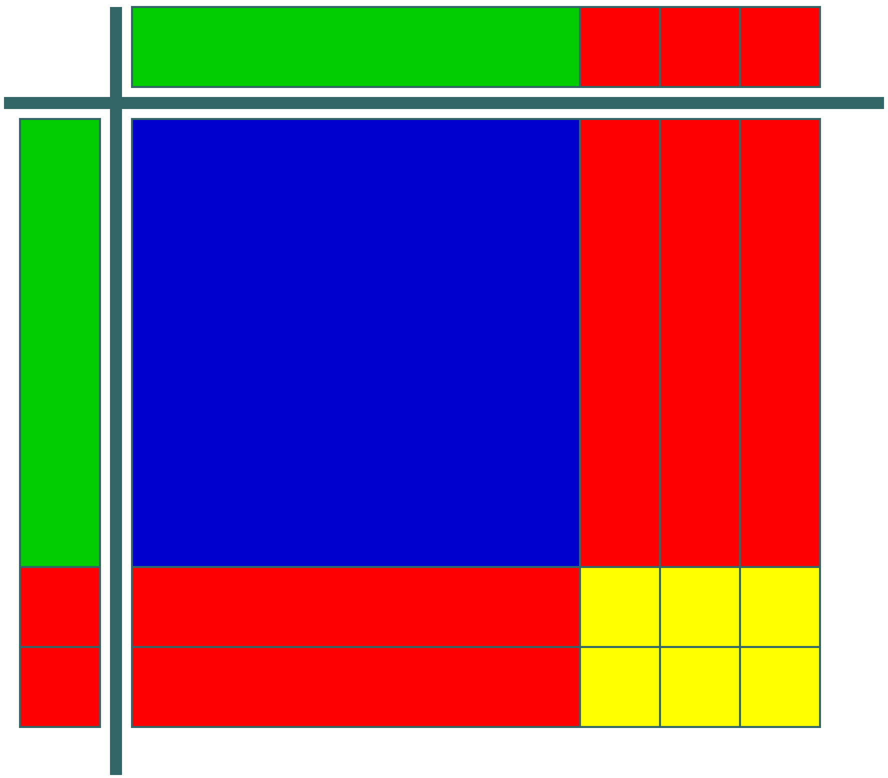
Factoring Polynomials

$x^2 + 6x + 8$



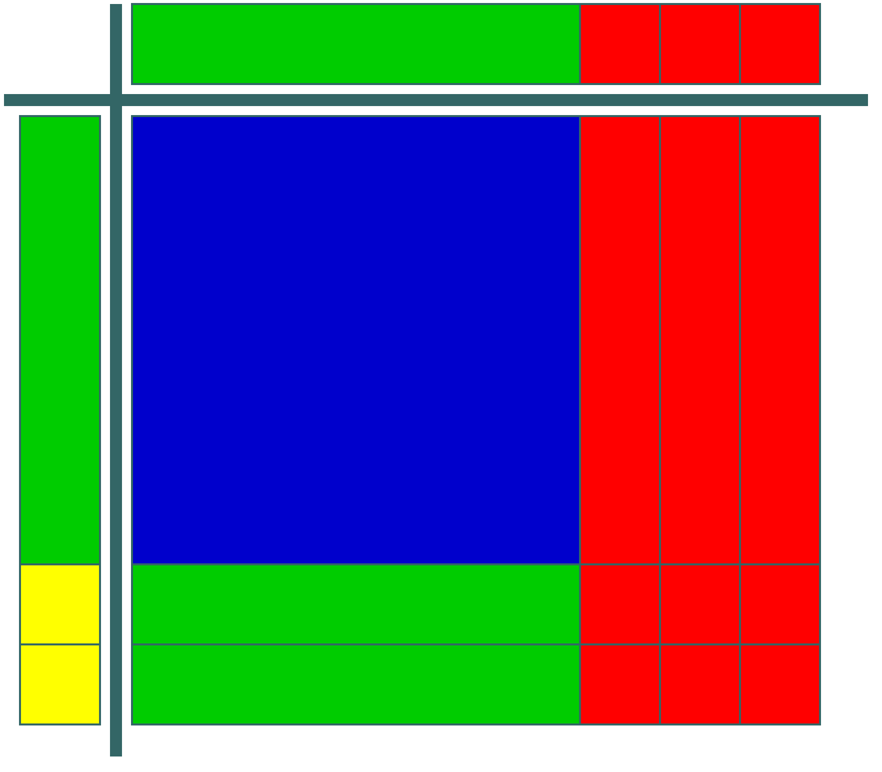
Factoring Polynomials

$x^2 - 5x + 6$



Factoring Polynomials

$x^2 - x - 6$



Dividing Polynomials

- Algebra tiles can be used to divide polynomials.
- Use tiles and frame to represent problem. Dividend should form array inside frame. Divisor will form one of the dimensions (one side) of the frame.
- Be prepared to use zero pairs in the dividend.

Dividing Polynomials

$$\frac{x^2 + 7x + 6}{x + 1}$$

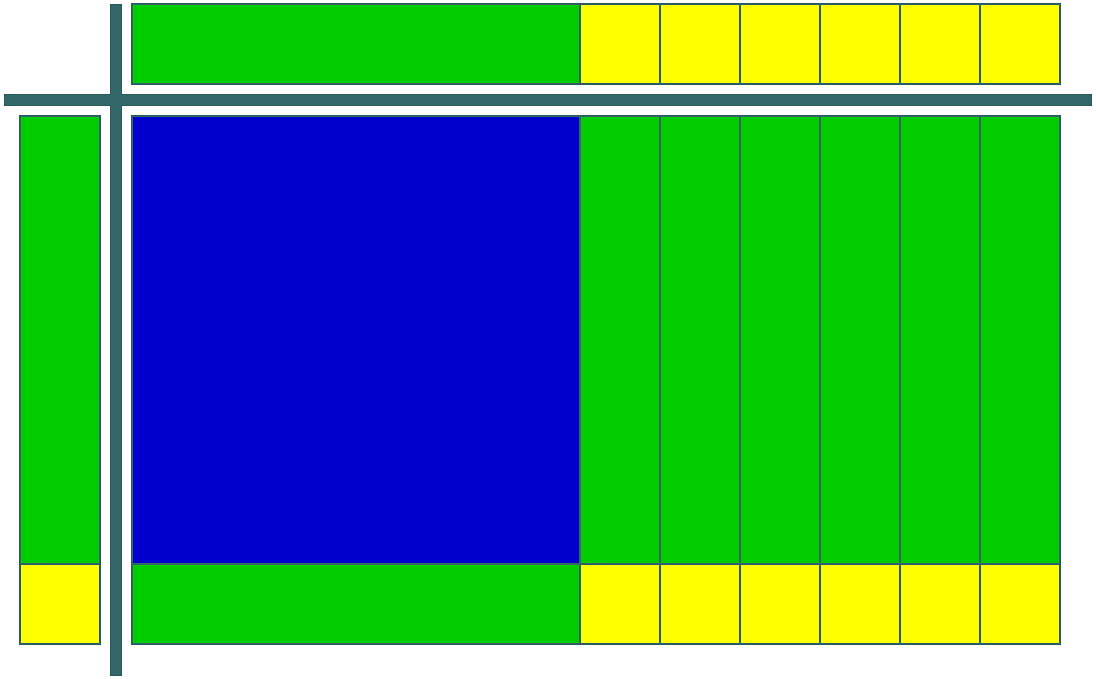
$$\frac{2x^2 + 5x - 3}{x + 3}$$

$$\frac{x^2 - x - 2}{x - 2}$$

$$\frac{x^2 + x - 6}{x + 3}$$

Dividing Polynomials

$$\frac{x^2 + 7x + 6}{x + 1}$$



Conclusion

“Polynomials are unlike the other “numbers” students learn how to add, subtract, multiply, and divide. They are not “counting” numbers. Giving polynomials a concrete reference (tiles) makes them real.”

David A. Reid, Acadia University