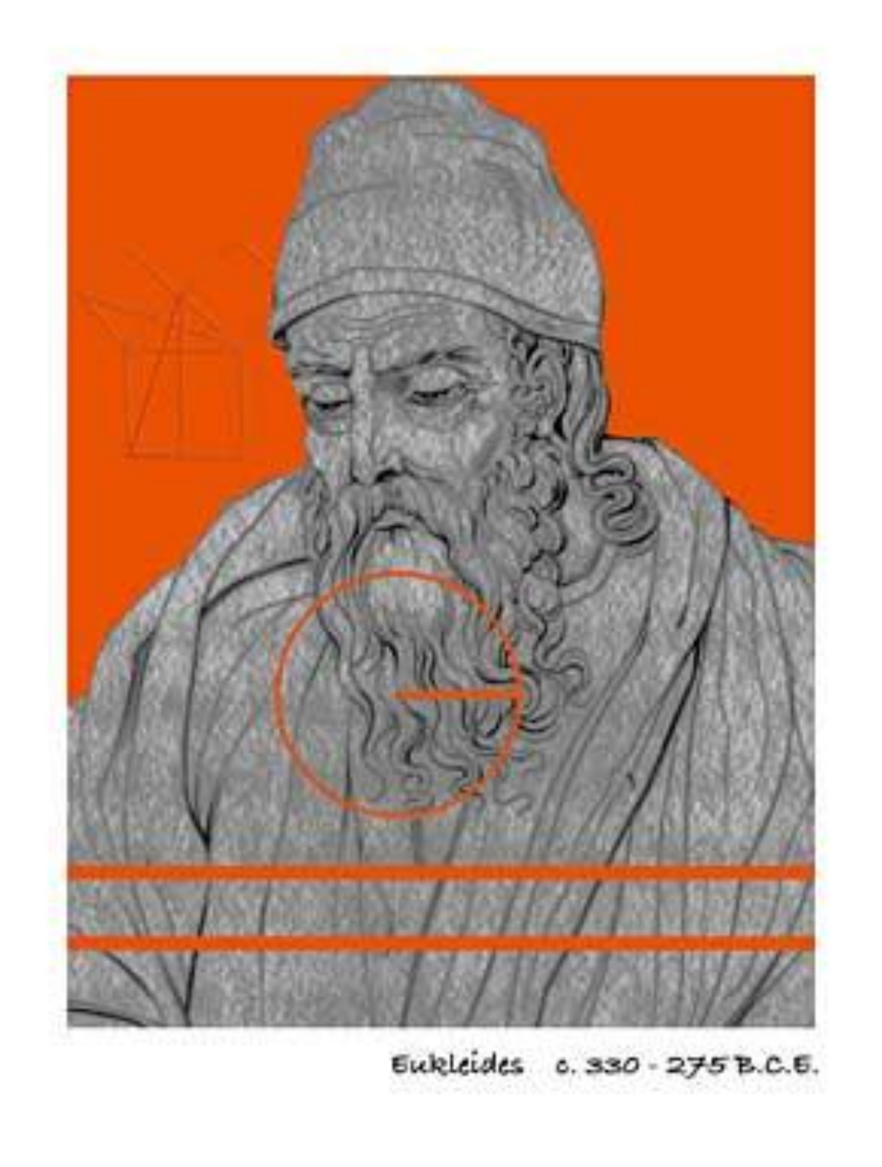
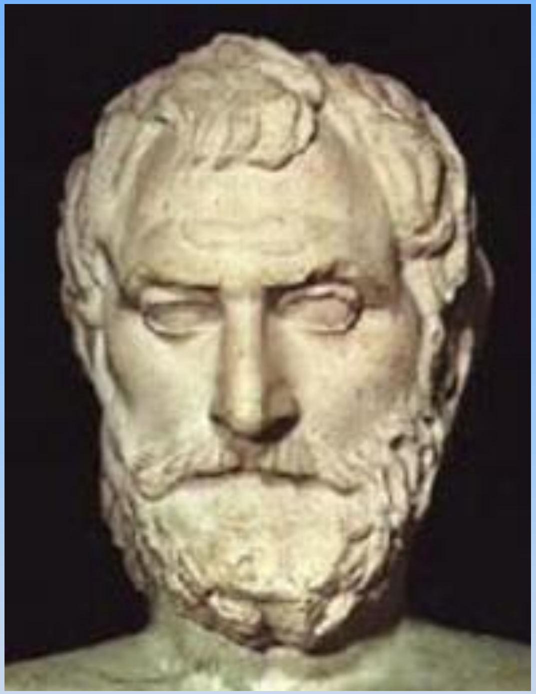
Euclidean Geometry Proofs



History

- Thales (600 BC)
  - First to turn geometry into a logical discipline.
  - Described as the first Greek philosopher and the father of geometry as a deductive study.
  - Relied on rational thought rather than mythology to explain the world around him.
- Pythagoreans and other Greeks continued this rational train of thought.



History

- By the time of Euclid many things had been proved by Greek mathematicians.
  - However, these proofs were disorganized, each one starting from its own set of assumptions.
- Euclid organized many of these proofs and more that he came up with in his work *Elements*.
- Euclid is generally considered a great mathematician. But, In fact he was not. He was considered the best school teacher in history as written by Van der Waerden.
- He prepared his textbook that was for adult students, and made it so wonderful that over a thousand editions were made after 1492 when it was first printed.

School of Athens

- **The most prominent persons are Platon, Aristotle, Socrates, Zoroaster, Pythagoras, Ptolemy. Raphael, Sodoma and Michelangelo are also present.**

Why do we have to learn this?

A student questioned Euclid and what they would get from learning the subject.

Then Euclid ordered someone to give him a penny.

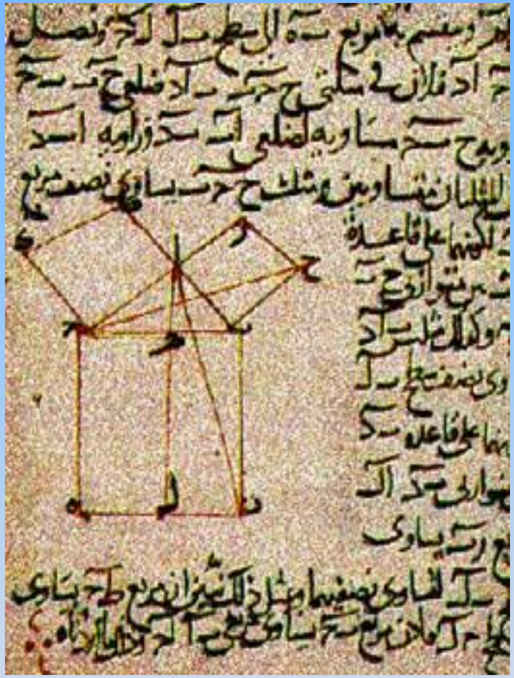
“since he must gain from what he learns”

Euclid

- We don’t know when he was born or died. We know that he was younger than students of Plato, but older than Archimedes, and that is all.
- One tells that when Ptolemy asked him about a short or quick way to learn geometry.
- Euclid answered there is no king’s road in geometry.

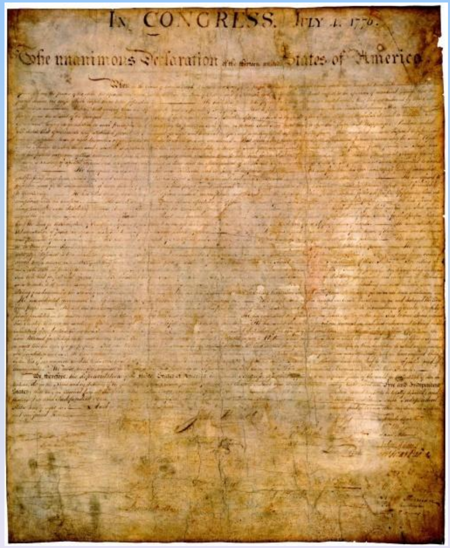
The Elements

- Composed of thirteen parts or “books” (probably long papyrus scrolls)
  - Books I – IV & VI are on plane geometry.
  - Books V & X are about magnitudes and ratios.
  - Books VII – IX are about whole numbers.
  - Books XI – XIII are about solid geometry.
- These thirteen books contained a total of 465 “propositions” or theorems.
  - Had a figure corresponding to each proposition followed by a careful proof.
  - The proof then ends with a restatement of the original proposition to be proved.



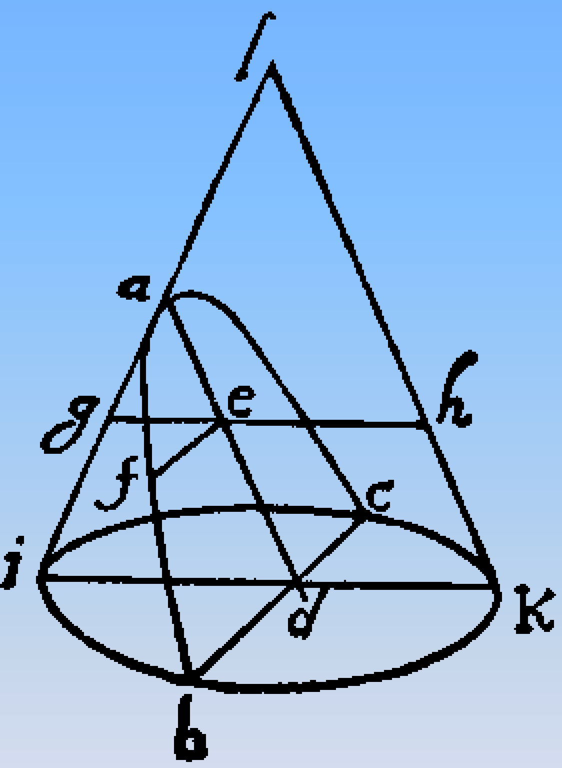
Relevance

- No other book except the *Bible* has been so widely translated and circulated.
- Early copy stored at the Vatican Library.
- Euclid's *Elements* was not just a mathematical step forward but was also a step forward in logical thinking.
- Things based on or influenced by the ideas of Euclid's *Elements*:
    - Descartes philosophical method.
        - Moving from basic principles to complex conclusions.
    - Newton and Spinoza used the form of Euclid's *Elements* to present their ideas.
    - Abraham Lincoln carried a copy of *Elements* with him in order to be a better lawyer.
    - The Declaration of Independence is based on "self evident" axioms used to prove the colonies are justified in forming the United States of America.



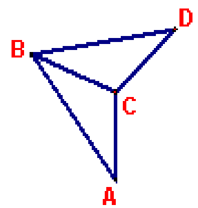
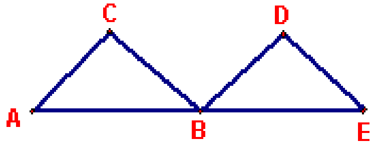
Euclid Today

- Today, a modified form of Euclid's *Elements* is used as the curriculum for sophomores in high school although the logic is slightly de-emphasized.
- The logic of Euclid's *Elements* is valid in many parts of modern life:
  - Such as collective bargaining agreements, computer systems, software development, and dealing with social-political arguments.
- Basically, Euclid developed a way of organizing ideas in a logical manner that is still relevant today.

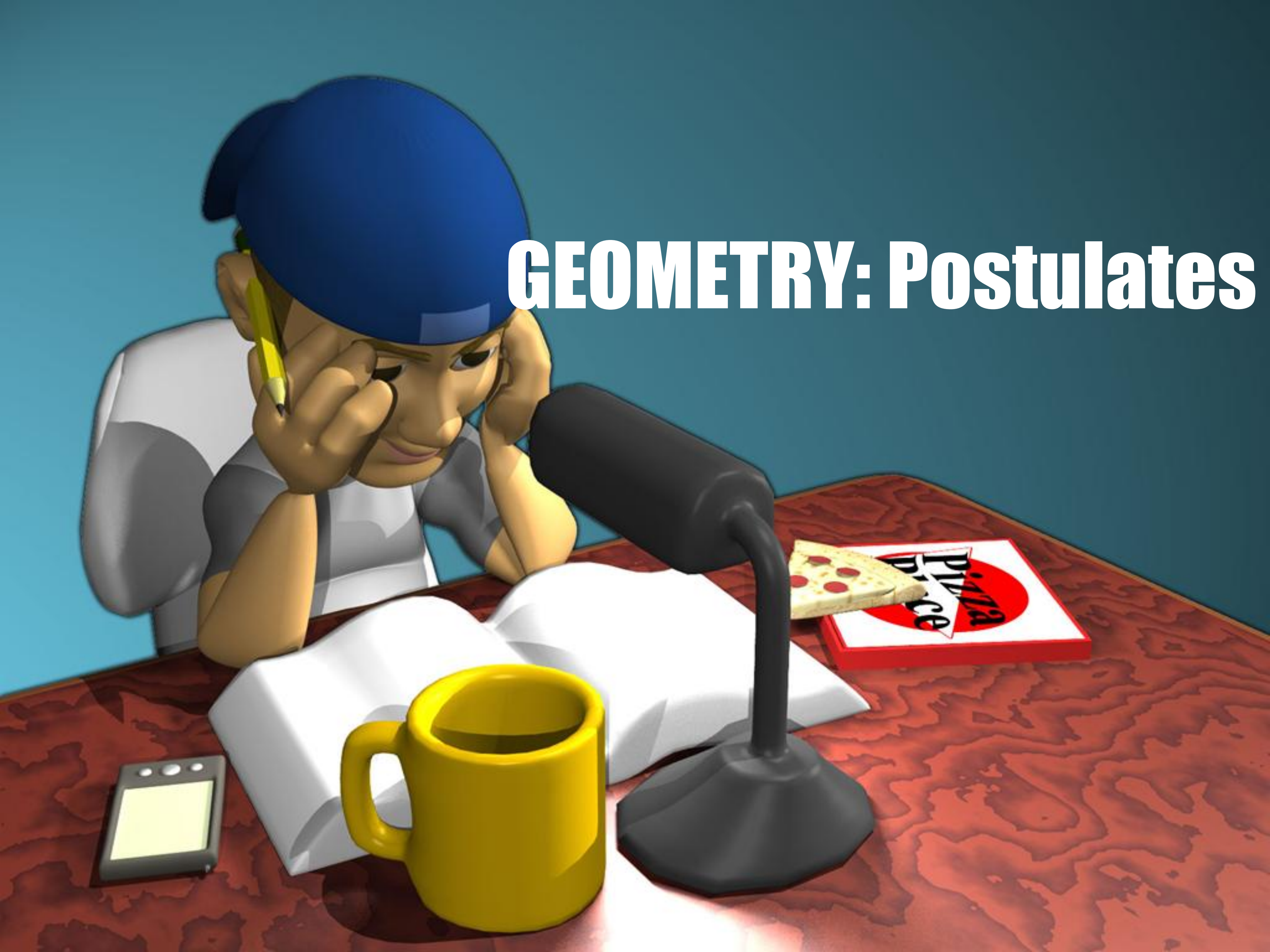


GEOMETRY PROOFS

GEOMERTRY : PROPERTIES

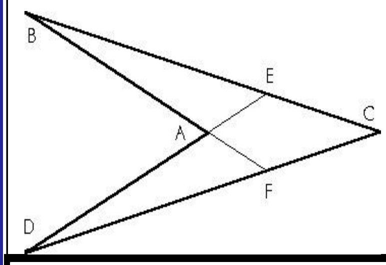
A.	**REFLEXIVE PROPERTY-**	*A quantity is congruent (equal) to itself*
	Statements	**Reasons**
	1. $\overline{BC} \cong \overline{BC}$	1. *Reflexive property*
B.	**TRANSITIVE PROPERTY-**	*If a=b and b=c, then a=c*
	Statements	**Reasons**
Given: $\overline{AC} \cong \overline{CB}$ and $\overline{CB} \cong \overline{DB}$ 	1. $\overline{AC} \cong \overline{CB}$ and $\overline{CB} \cong \overline{DB}$ 2. $\overline{AC} \cong \overline{DB}$	1. *Given* 2. *Transitive property*
C.	**SYMMETRIC PROPERTY-**	*If a=b, then b=a*

GEOMETRY: Postulates



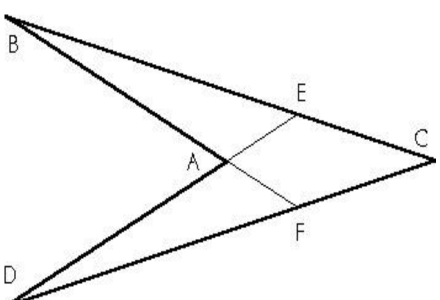
A. **ADDITION POSTULATE-** *If equal quantities are added to equal quantities, the sums are equal*

	Statements	**Reasons**
Given: $\overline{BE} \cong \overline{DF}$ and $\overline{EC} \cong \overline{FC}$	1. $\overline{BE} \cong \overline{DF}$ and $\overline{EC} \cong \overline{FC}$ 2. $\overline{BE} + \overline{EC} \cong \overline{DF} + \overline{FC}$ $\overline{BC} \cong \overline{DC}$	1. *Given* 2. *Addition postulate*



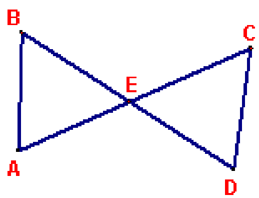
B. **SUBTRACTION POSTULATE-** *If equal quantities are subtracted from equal quantities, the differences are equal*

	Statements	**Reasons**
Given: $\overline{BC} \cong \overline{DC}$ and $\overline{EC} \cong FC$	1. $\overline{BC} \cong \overline{DC}$ and $\overline{EC} \cong \overline{FC}$ 2. $\overline{BC} - \overline{EC} \cong \overline{DC} - \overline{FC}$ $\overline{BE} \cong \overline{DF}$	1. *Given* 2. *Subtraction postulate*

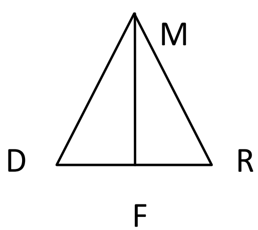


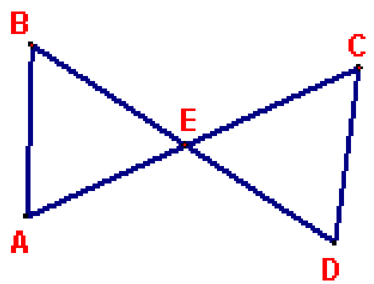
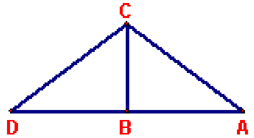
GEOMERTRY : DEFINITIONS

A. **DEFINITION OF MIDPOINT-** *A point on a line segment that divides the segment into two congruent segments*

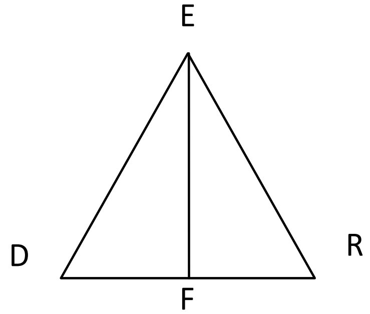
	Statements	**Reasons**
Given: E is the midpoint of $\overline{BD}$ 	1.E is the *midpoint* of $\overline{BD}$ 2. $\overline{BE} \cong \overline{DE}$	*1. Given* *2. Definition of midpoint*

B. **DEFINITION OF MEDIAN-** *A line segment that joins any vertex of the triangle to the **midpoint** of the opposite side.*

	Statements	**Reasons**
Given: $\overline{MF}$ is the median of $\overline{DR}$ 	1. $\overline{MF}$ is the median of $\overline{DR}$ 2. F is the midpoint of $\overline{DR}$ 3. $\overline{DF} \cong \overline{RF}$	*1. Given* *2. Definition of median* *3. Definition of midpoint*

	C. **DEFINITION OF VERTICAL ANGLES-**	*When two lines intersect vertical angles are formed.*
	Statements	**Reasons**
	1. $\angle BEA$ and $\angle DEC$ are vertical angles 2. $\angle BEA \cong \angle DEC$	*1. If two lines intersect then vertical angles are formed* *2. Vertical angles are congruent*
	D. **DEFINITION OF PERPENDICULAR LINES-**	*two lines that intersect to form* ***right angles***
	Statements	**Reasons**
Given: $\overline{CB} \perp \overline{DA}$ 	1. $\overline{CB} \perp \overline{DA}$ 2. $\angle CBD$ and $\angle CBA$ are right angles 3. $\angle CBD \cong \angle CBA$	*1. Given* *2. Definition of perpendicular lines* *3. All right angles are congruent*

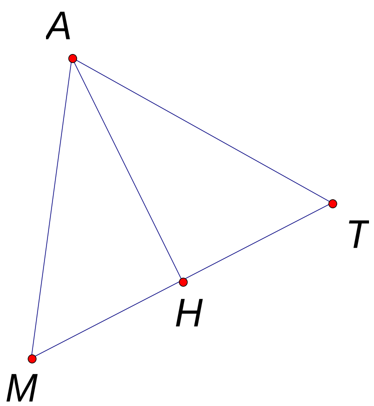
E. **DEFINITION OF ALTITUDE-** *A line segment drawn from any vertex of the triangle, **perpendicular** to and ending in the line that contains the opposite side.*

	Statements	**Reasons**
Given: $\overline{EF}$ is the altitude of $\triangle$DER 	1. $\overline{EF}$ is the altitude of $\triangle$DER 2. $\overline{EF} \perp \overline{DR}$ 3. $\angle EFD$ and $\angle$EFR are right angles 4. $\angle EFD \cong \angle EFR$	*1. Given* *2. Definition of altitude* *3. Definition of perpendicular lines* *4. All right angles are congruent*

F. **<u>DEFINITION OF ANGLE BISECTOR-</u>** *A ray whose endpoint is the vertex of the angle, and that divides that angle into two congruent angles.*

<u>Given:</u>

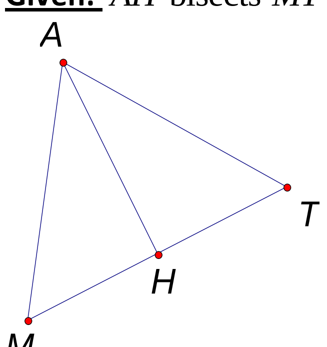
$\overline{AH}$ bisects $\angle MAT$



	<u>Statements</u>	**<u>Reasons</u>**
	1. $\overline{AH}$ bisects $\angle MAT$	*1. Given*
	2. $\angle MAT \cong \angle TAH$	*2. Definition of <u>angle bisector</u>*

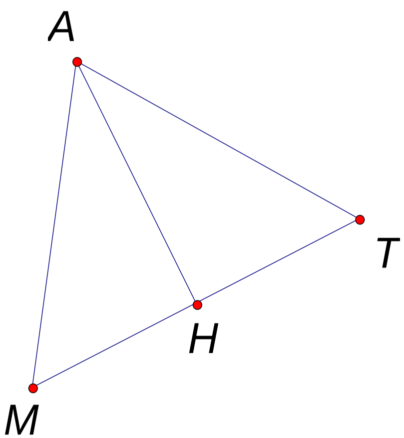
G. **<u>DEFINITION OF SEGMENT BISECTOR-</u>** *Any line, or subset of a line, that intersects the segment at its midpoint.*

<u>Given:</u> $\overline{AH}$ bisects $\overline{MT}$



	<u>Statements</u>	**<u>Reasons</u>**
	1. $\overline{AH}$ bisects $\overline{MT}$	*1. Given*
	2. $\overline{MH} \cong \overline{TH}$	*2. Definition of <u>segment bisector</u>*

H. **<u>DEFINITION OF PERPENDICULAR BISECTOR-</u>** *Any line, or subset of a line, that is perpendicular to the line segment at its midpoint.*

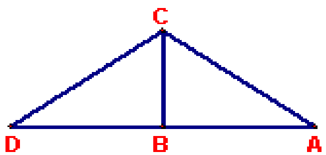
	<u>Statements</u>	**<u>Reasons</u>**
<u>Given:</u> $\overline{AH}$ is the perpendicular bisector of $\overline{MT}$ 	**1.** $\overline{AH}$ is the perpendicular bisector of $\overline{MT}$ **2.** $\angle AHM$ and $\angle AHT$ are right angles **3.** $\angle AHM \cong \angle AHT$ **4.** $\overline{MH} \cong \overline{TH}$	*1. Given* *2. Definition of <u>perpendicular</u> lines* *3. All <u>right angles</u> are congruent* *4. Definition of <u>segment bisector</u>*

GEOMETRY

THEOREMS

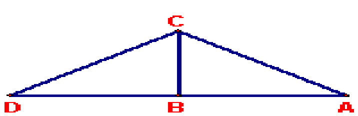
A. **BASE ANGLE THEOREM- (ISOSCELES TRIANGLE)**

If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

	Statements	**Reasons**
Given: $\overline{CD} \cong \overline{CA}$ 	1. $\overline{CD} \cong \overline{CA}$ 2. $\angle A \cong \angle D$	*1. Given* 2. *If two sides of a triangle are congruent, then the angles opposite these sides are congruent.*

B. **CONVERSE OF THE BASE ANGLE THEOREM – (ISOSCELES TRIANGLE)**

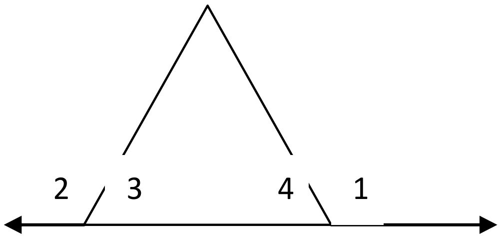
If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

	Statements	**Reasons**
Given: $\angle A \cong \angle D$ 	1. $\angle A \cong \angle D$ 2. $\overline{CD} \cong \overline{CA}$	*1. Given* 2. *If two angles of a triangle are congruent, then the sides opposite these angles are congruent.*

C. **CONGRUENT SUPPLEMENTS-** *If two angles are supplementary to the same angle (or to congruent angles), then the two angles are congruent.*

Or

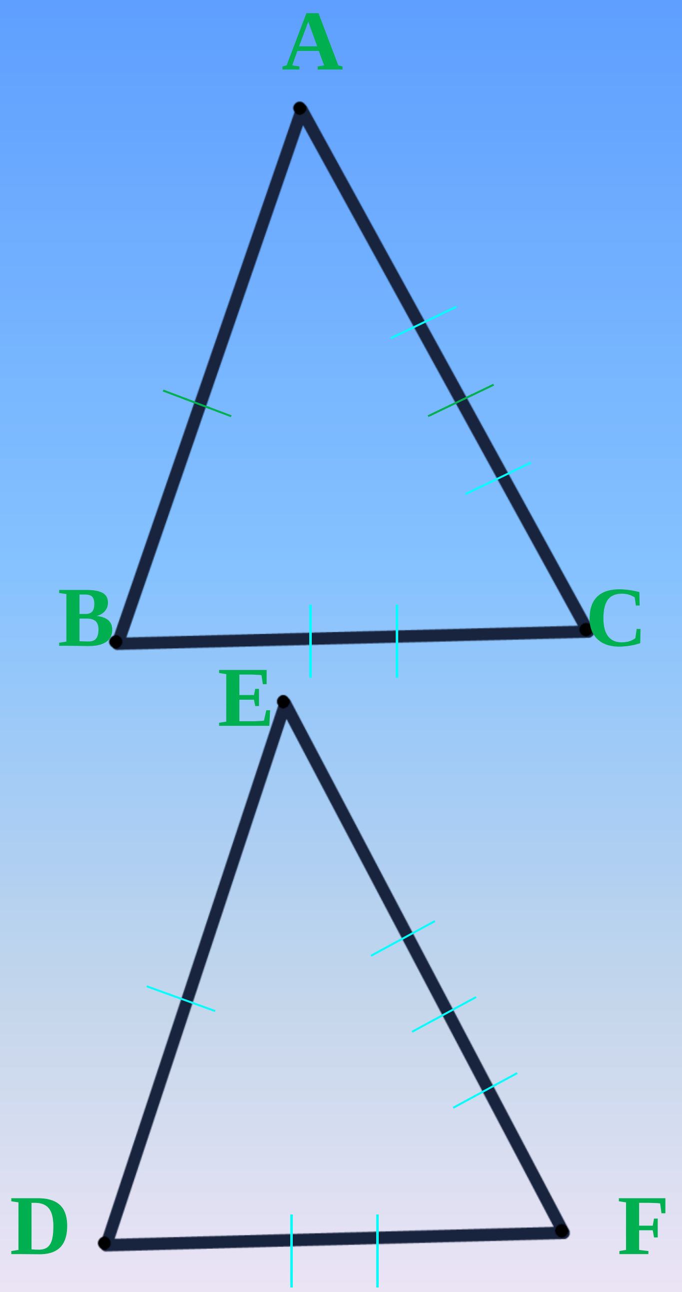
"Supplements of congruent angles are congruent"

	Statements	**Reasons**
Given: $\angle 2 \cong \angle 1$ 	1. $\angle 2 \cong \angle 1$ 2. $\angle 2$ is supplementary to $\angle 3$ $\angle 1$ is supplementary to $\angle 4$ 3. $\angle 3 \cong \angle 4$	*1. Given* *2. If two angles for a linear pair, then they are supplementary* 3. *Supplements of congruent angles are congruent*

GEOMERTRY : Postulates used to prove triangles are congruent.

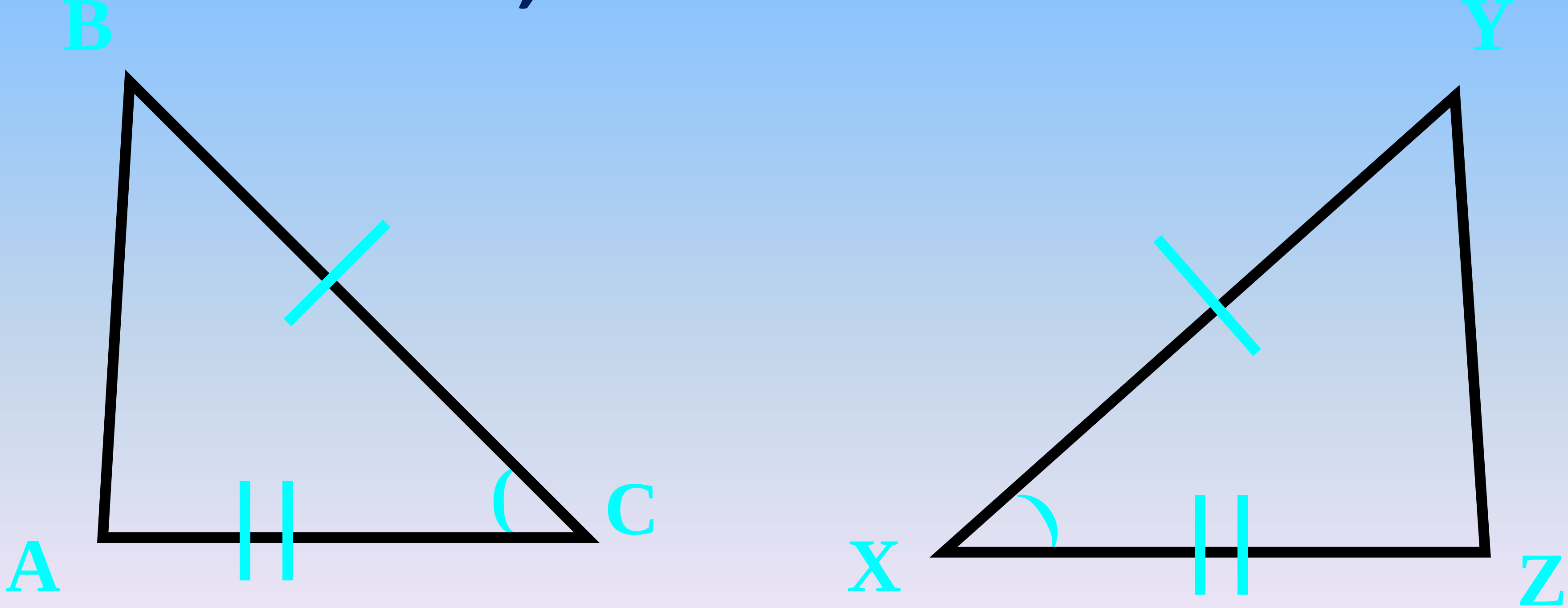
Side-Side-Side (SSS) Postulate

If 3 sides of one Δ are ≅ to 3 sides of another Δ, then the Δs are ≅.

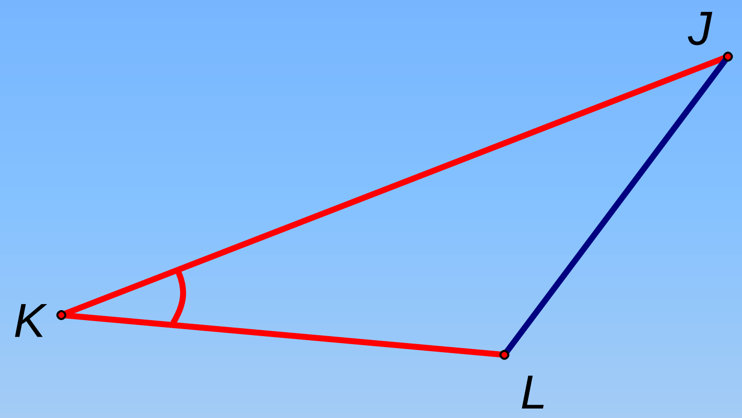


Side-Angle-Side (SAS) Postulate

If 2 sides and the included ∠ of one Δ are ≅ to 2 sides and the included ∠ of another Δ, then the 2 Δs are ≅.



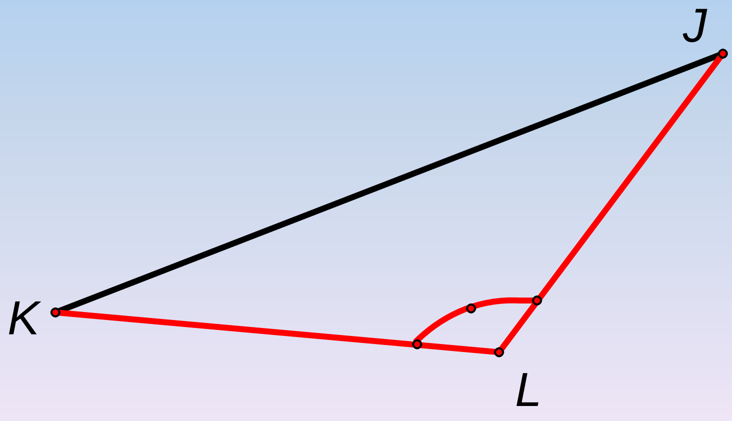
Definition – Included Angle



∠K is the angle between JK and KL. It is called the *included angle* of sides JK and KL.

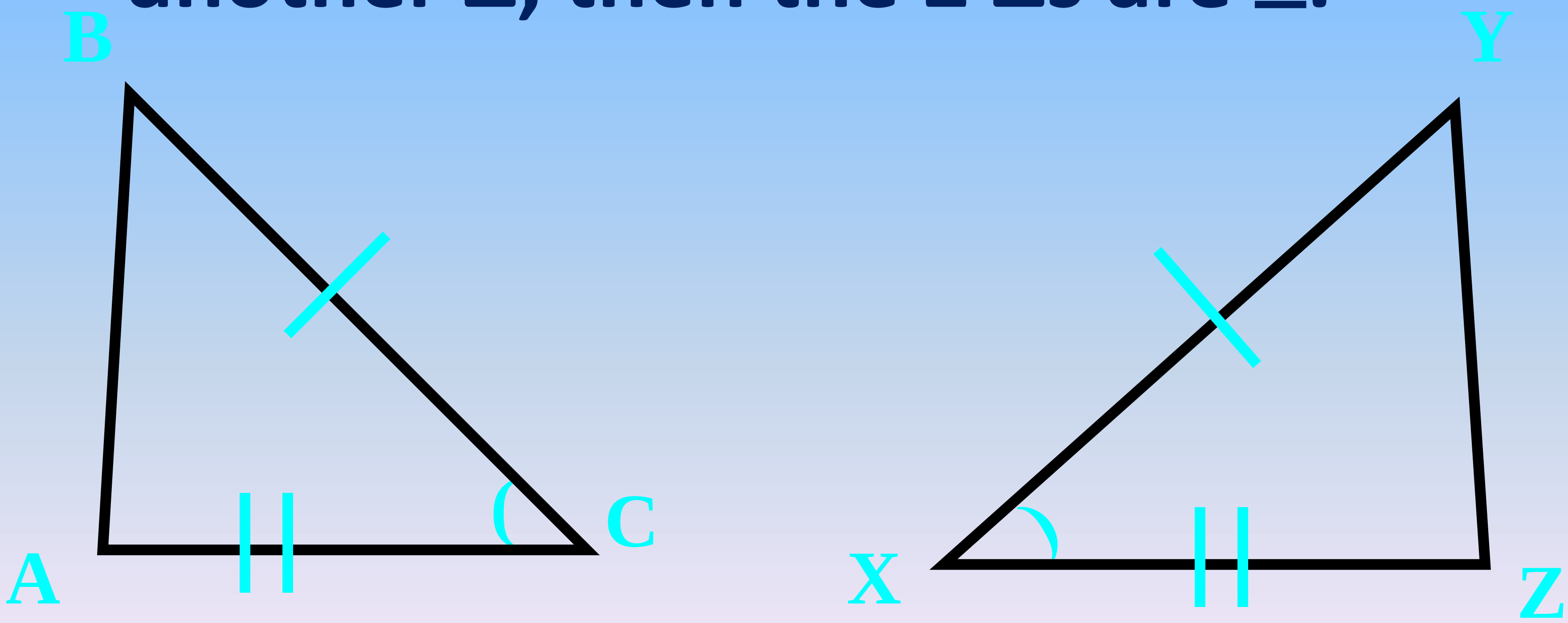
What is the included angle for sides KL and JL?

∠L



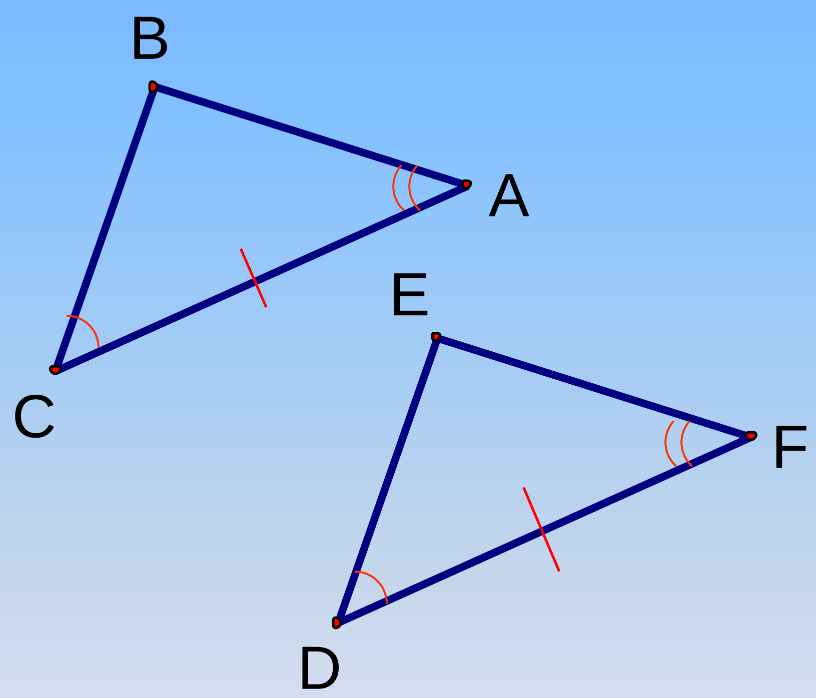
Side-Angle-Side (SAS) Postulate

If 2 sides and the included ∠ of one Δ are ≅ to 2 sides and the included ∠ of another Δ, then the 2 Δs are ≅.



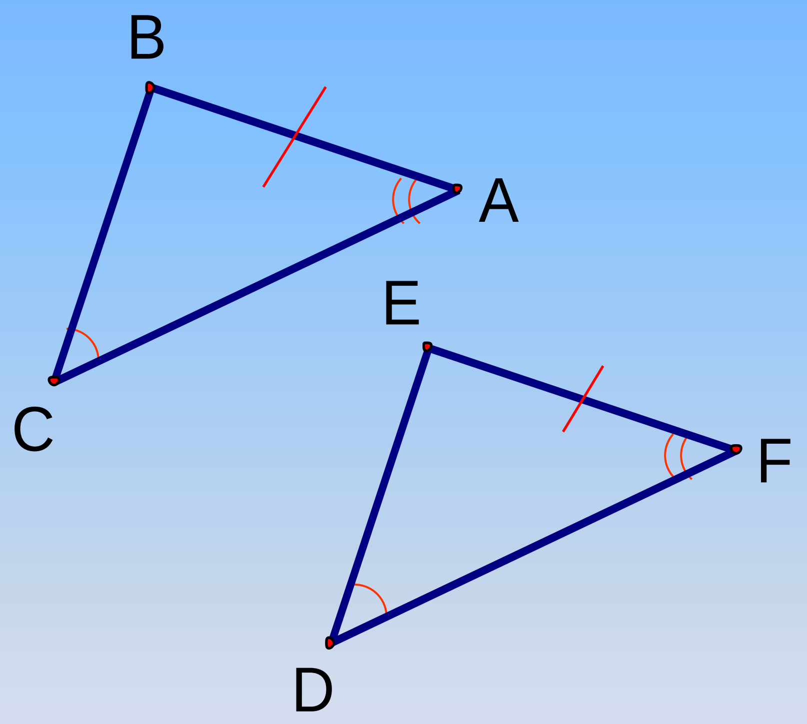
Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.

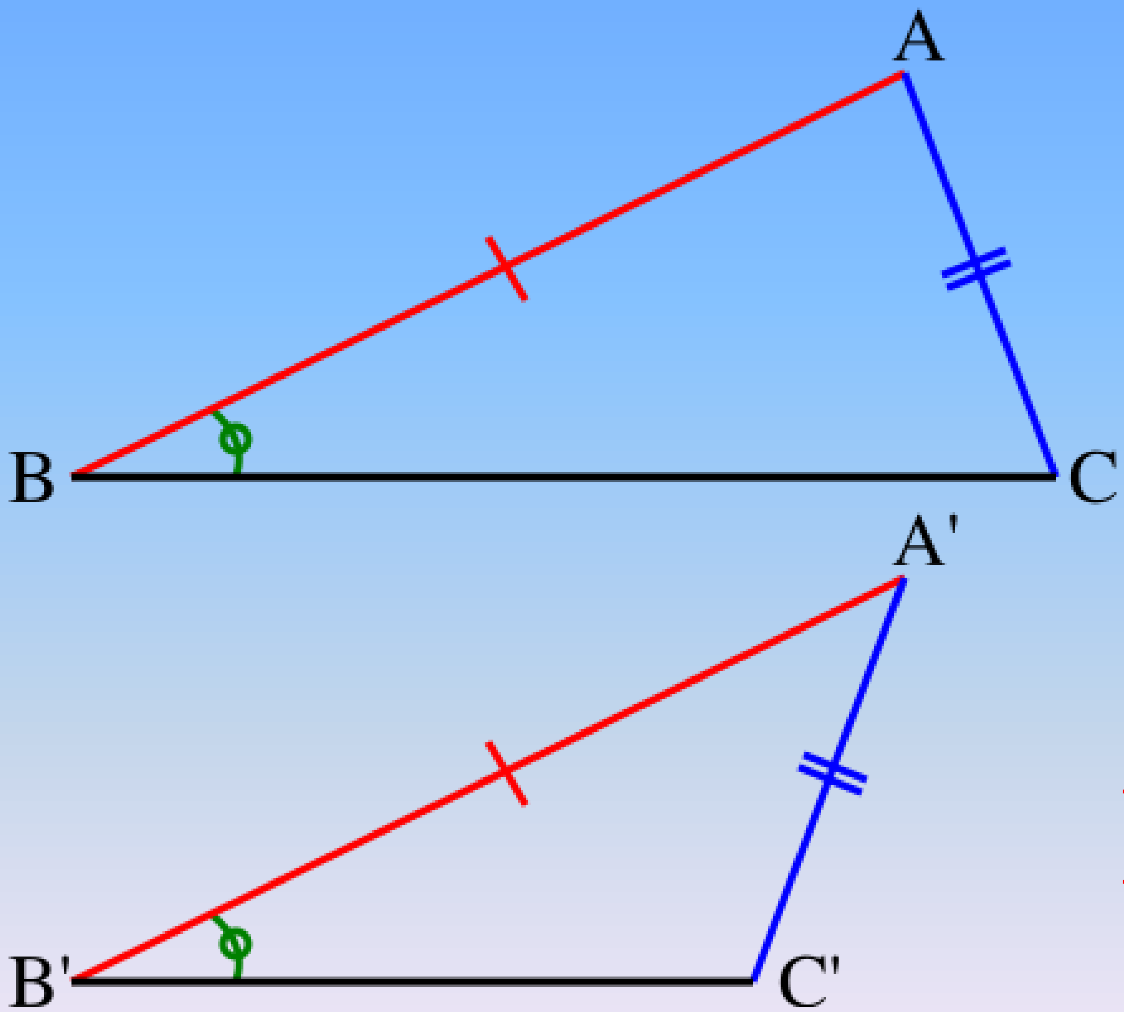


Angle-Angle-Side (AAS) Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the triangles are congruent.



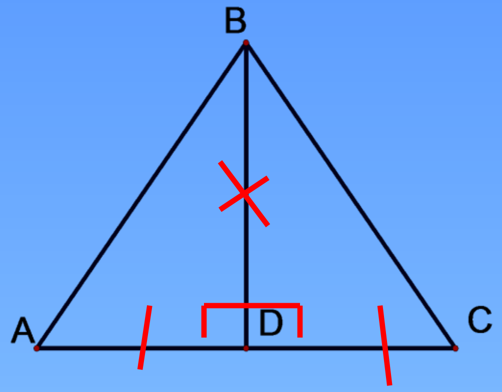
SSA?



NOPE!

Given: BD ⊥ AC and AD ≅ DC

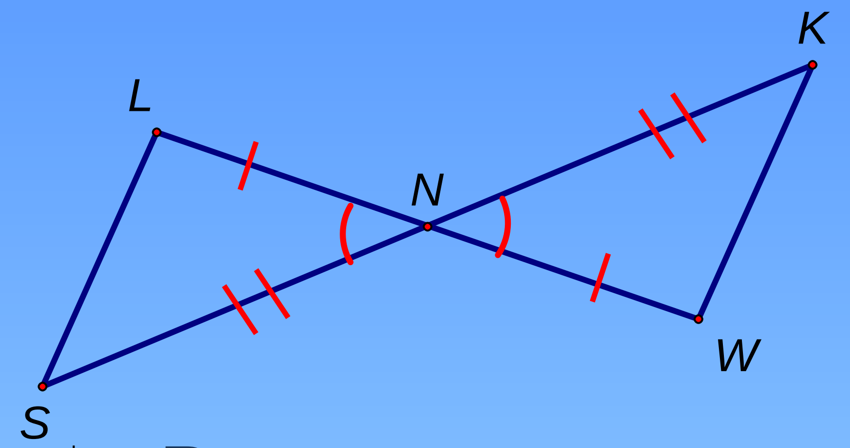
Prove: Δ ADB ≅ Δ CDB.



Statements	**Reasons**
1. BD ⊥ AC; AD ≅ DC	1. Given
2. ∠ADB & ∠CDB are right ∠s	2. Definition of ⊥ lines
3. ∠ADB ≅ ∠ CDB	3. All right angles are ≅.
4. DB ≅ DB	4. Reflexive Property
5. Δ ADB ≅ Δ CDB	5. SAS ≅ SAS

Given: N is the midpoint of LW
N is the midpoint of SK

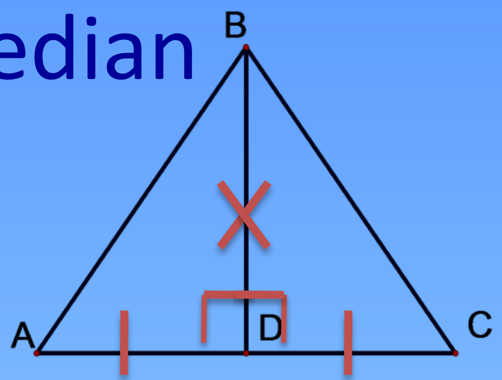
Prove: $\triangle LNS \cong \triangle WNK$



Statements	Reasons
1. N is the midpoint of LW N is the midpoint of SK	1. Given
2. $\overline{LN} \cong \overline{NW}$, $\overline{SN} \cong \overline{NK}$	2. Definition of Midpoint
3. $\angle LNS$ & $\angle WNK$ are vertical angles	3. If two lines intersect, then vertical angles are formed.
4. $\angle LNS \cong \angle WNK$	4. Vertical Angles are congruent
5. $\triangle LNS \cong \triangle WNK$	5. SAS Postulate

Given: BD is an altitude; BD is a median

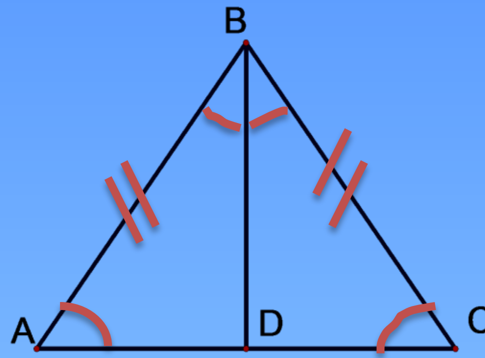
Prove: Δ ADB ≅ Δ CDB.



Statements	Reasons
1. BD is a altitude	1. Given
2. DB ⊥ AC	2. Definition of Altitude
3. ∠ADB & ∠CDB are right ∠s	3. Definition of ⊥ lines.
4. ∠ADB ≅ ∠ CDB	4. All right angles are ≅.
5. BD is a median	5. Given
6. D is a midpoint of AC	6. Definition of median
7. AD ≅ DC	7. Definition of midpoint
8. BD ≅ BD	8. Reflexive
9. Δ ADB ≅ Δ CDB	9. SAS

Given: **BD bisects ∠ABC; AB ≅ BC**

Prove: Δ ADB ≅ Δ CDB.

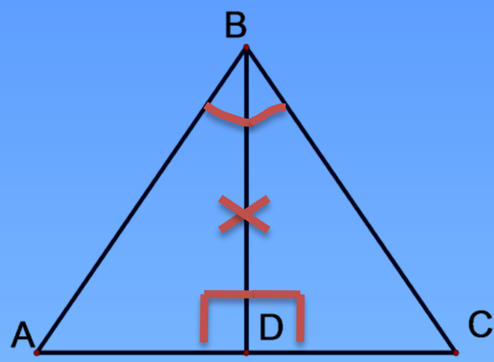


Statements	**Reasons**
1. BD bisects ∠ABC	**1. Given**
2. ∠ABD ≅ ∠ CBD	**2. Definition of angle bisector**
3. AB ≅ BC	**3. Given**
4. ∠BAD ≅ ∠ BCD	**4. If two sides of a triangle are ≅ then the angles opposite those sides are ≅.**
5. Δ ADB ≅ Δ CDB	**5. SAS**

Given: BD bisects ∠ABC ;

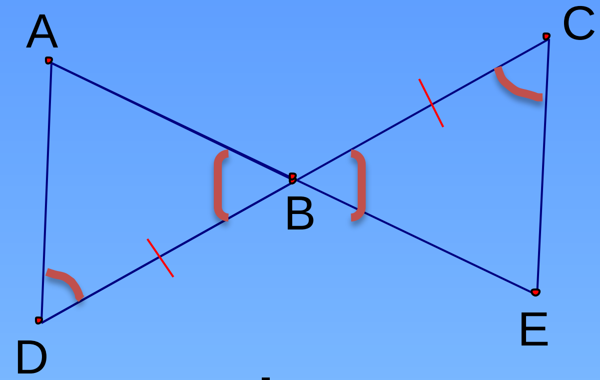
BD is an altitude

Prove: Δ ADB ≅ Δ CDB.



Statements	Reasons
1. BD is a altitude	1. Given
2. DB ⊥ AC	2. Definition of Altitude
3.∠ADB & ∠CDB are right ∠s	3. Definition of ⊥ lines.
4.∠ADB≅ ∠ CDB	4. All right angles are ≅.
5. BD bisects ∠ABC	5. Given
6. ∠ABD ≅ ∠CBD	6. Definition of angle bisector
7. BD ≅ BD	7. Reflexive
8. Δ ADB ≅ Δ CDB	8. ASA

Proof:



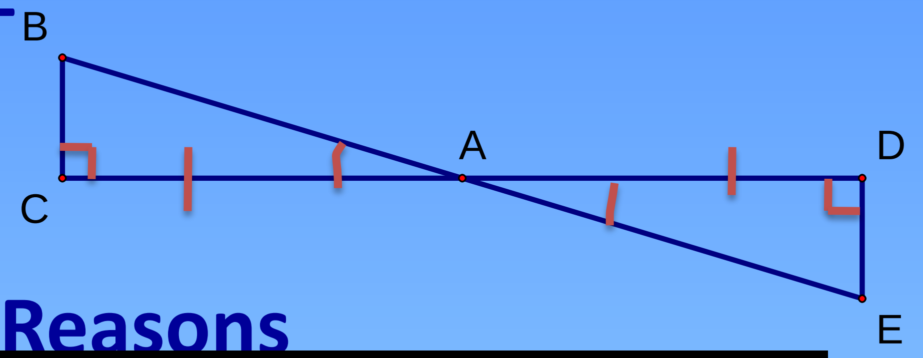
Given: AD ∥ EC, BD ≅ BC
Prove: ΔABD ≅ ΔEBC
and **AB ≅ BE**

Statements:	Reasons:
1. BD ≅ BC	1. Given
2. ∠ABD & ∠EBC are vertical angles	2. Intersecting lines form vertical angles.
3. ∠ABD ≅ ∠EBC	3. Vertical angles are congruent.
4. AD ∥ EC	4. Given
5. ∠D ≅ ∠C	5. If two ∥ lines are cut by a transversal, then Alternate Interior Angles are congruent
6. ΔABD ≅ ΔEBC	6. ASA
7. AB ≅ BE	7. CPCTC

Given: $BC \perp CD$; $CD \perp DE$

BE bisects CD

Prove: $\Delta BCA \cong \Delta EDA$.



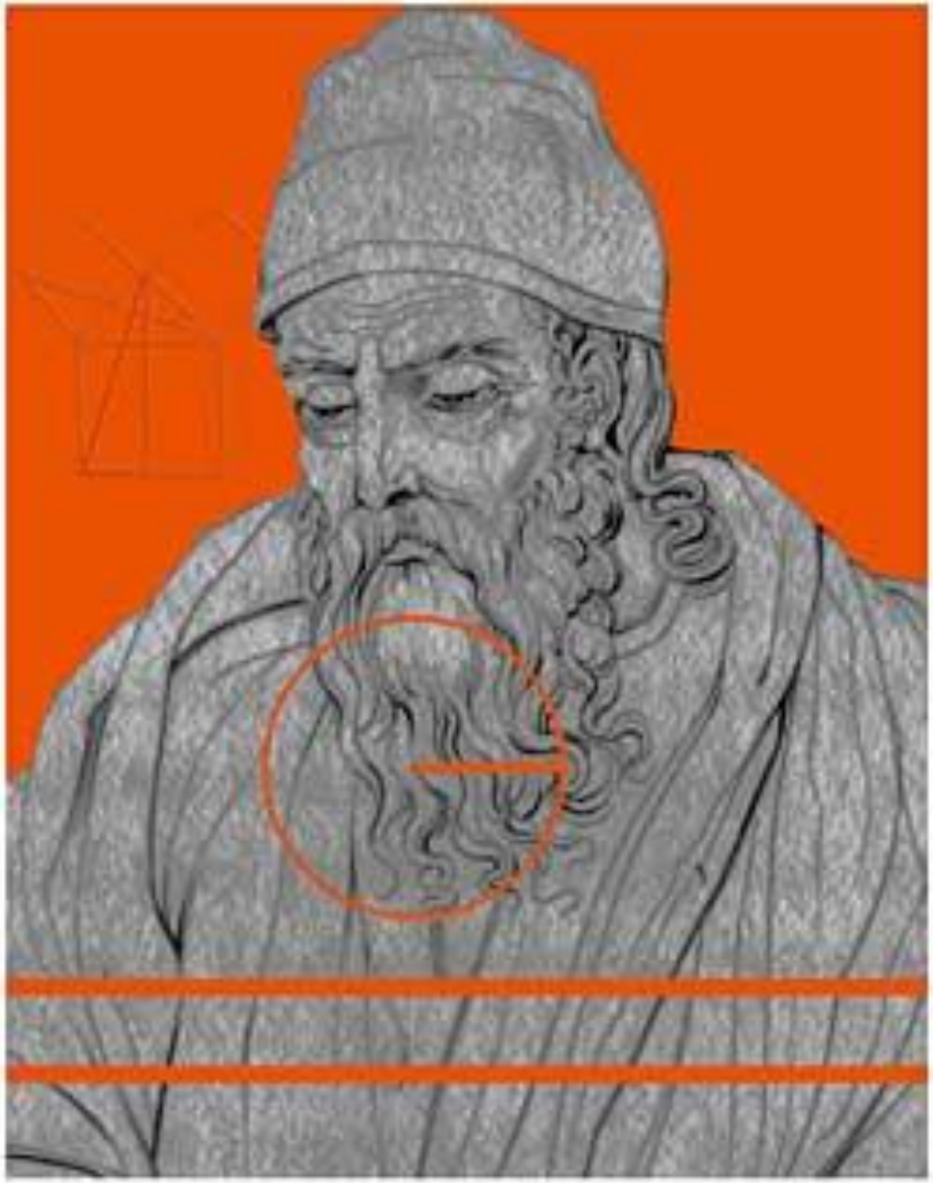
Statements	Reasons
1. $BC \perp CD$; $CD \perp DE$	1. Given
2. $\angle ACB$ & $\angle ADE$ are right $\angle$s	2. Definition of $\perp$ lines.
3. $\angle ACB \cong \angle ADE$	3. All right angles are $\cong$.
4. BE bisects CD	4. Given
5. $AC \cong AD$	5. Definition of segment bisector
6. $\angle BAC$ & $\angle EAD$ are vertical $\angle$s	6. If two lines intersect vertical $\angle$s are formed.
7. $\angle BAC \cong \angle EAD$	7. Vertical angles are $\cong$.
8. $\Delta ADB \cong \Delta CDB$	8. ASA

Given: AB ⊥ BF ; EF ⊥ BF ; BD ≅ CF; AB ≅ EF

BD is an altitude

Prove: Δ ABC ≅ Δ EFD.

Statements	**Reasons**
1. AB ⊥ BF ; EF ⊥ BF	1. Given
2. ∠ABC & ∠EFD are right ∠s	2. Definition of ⊥ lines.
3. ∠ABC ≅ ∠ EFD	3. All right angles are ≅.
4. AB ≅ EF	4. Given
5. BD ≅ CF	5. Given
6. CD ≅ CD	6. Reflexive
7. BD – CD ≅ CF – CD BC ≅ DF	7. Subtraction Postulate
8. Δ ABC ≅ Δ EFD	8. ASA



Eukleides c. 330 - 275 B.C.E.

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