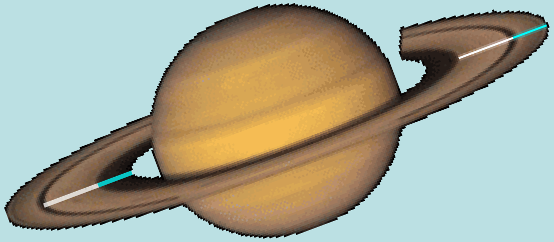
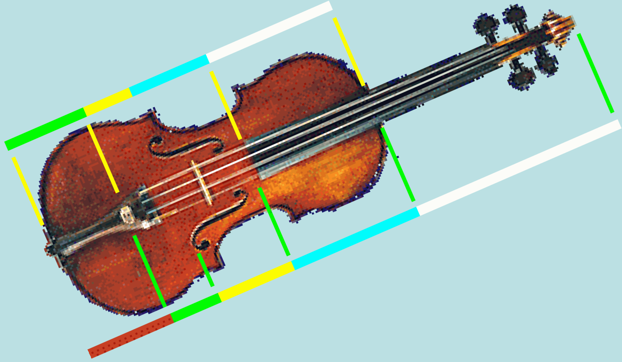


The Golden Ratio

Φ

The Divine Proportion



The Problem

There is a population of rabbits for which it is assumed:

1) In the first month there is just one new-born pair
2) New-born pairs become fertile from their second month on
3) Each month every fertile pair has a new pair
4) Rabbits never die.

In the first month there is just one new-born pair
New-born pairs become fertile from their second month on
Each month every fertile pair has a new pair
Rabbits never die.

Jan	Feb	Mar	April	May	Jun	July	Aug

In the first month there is just one new-born pair
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Each month every fertile pair has a new pair
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Jan	Feb	Mar	April	May	Jun	July	Aug
A							

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Jan	Feb	Mar	April	May	Jun	July	Aug
A	A						

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Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A B					

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Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A B	A B C				

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Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A B	A B C	A B C D D			

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Each month every fertile pair has a new pair
Rabbits never die.

Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A	A	A	A		
		B	B	B	B		
			C	C	C		
				D	D		
				D	D		
					E		
					E		
					E		

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Rabbits never die.

Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A B	A B C	A B C D D	A B C D D E E E	A B C D D EEE FFF FF	

In the first month there is just one new-born pair
New-born pairs become fertile from their second month on
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Jan	Feb	Mar	April	May	Jun	July	Aug
A	A	A	A	A	A	A	A
		B	B	B	B	B	B
			C	C	C	C	C
				D	D	D	DD
				D	D	D	EEE
					E	EEE	FFF
					E	FFF	FF
					E	FF	GGG
							GGG
							GG

Formula

- $F(1)=1$
- $F(2)=1$
- $F(3)=2$
- $F(4)=3$
- $F(5)=5$
- $F(n)=F(n-1)+F(n-2)$
- $F(n+1)=F(n)+F(n-1)$
- $F(n+2)=F(n+1)+F(n)$

What happens if we take the ratio of two consecutive Fibonacci numbers?

Let $x = \lim_{n\to\infty} \frac{F(n+1)}{F(n)}$

$$x = \lim_{n\to\infty} \frac{F(n)+F(n-1)}{F(n)}$$

$$x = \lim_{n\to\infty} \frac{F(n)}{F(n)} + \frac{F(n-1)}{F(n)}$$

$$x = \lim_{n\to\infty} 1 + \frac{1}{\frac{F(n)}{F(n-1)}}$$

$$x = \lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{1}{\frac{F(n)}{F(n-1)}}$$

$$x = 1 + \frac{1}{x}$$

$$x = 1 + \frac{1}{x}$$

$$x^2 = x + 1$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

Pick two numbers

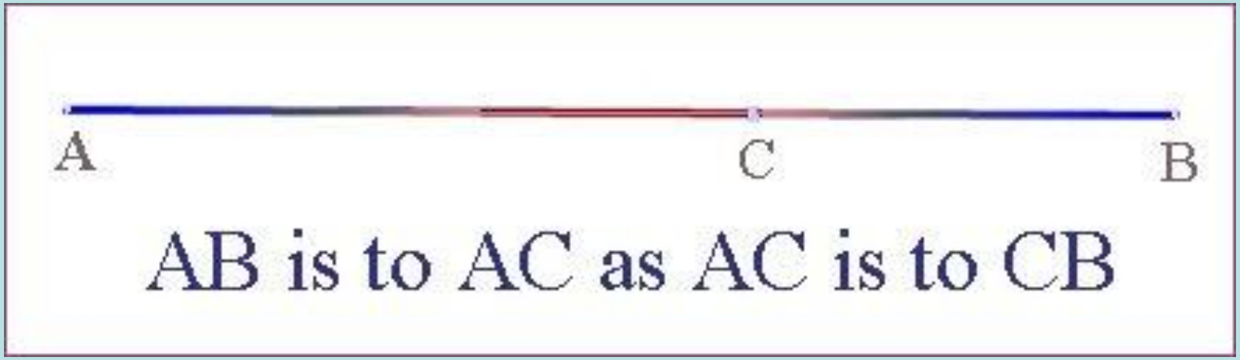
- a
- b
- a+b
- a+2b
- 2a+3b
- 3a+5b
- 5a+8b
- 8a+13b
- 13a+21b
- 21a+34b

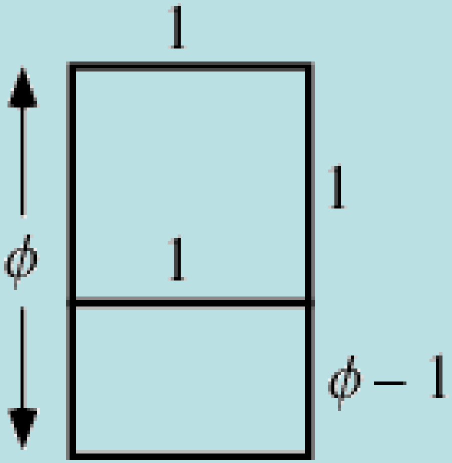
- The sum is 55a+88b = 11 (5a+8b)
- Fibonacci numbers

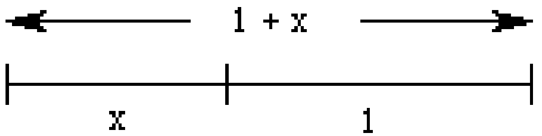
1 , 1, 2, 3, 5, 8, 13, 21, 34….

From Euclid (365-300 BCE) comes the following definition:

- If a straight line is cut in extreme and mean ratio, then as the whole is to the greater segment, the greater segment is the lesser segment.







$$\frac{x}{1} = \frac{1}{1+x}$$

$$x = \frac{-1+\sqrt{5}}{2}$$

$$\frac{longer}{smaller} = \frac{entire}{longer}$$

1.618...

.618...

1

Golden Ratio (phi)

1

x $\quad 1-x$

$$\frac{longer}{smaller} = \frac{entire}{longer}$$

$$\frac{x}{1-x} = \frac{1}{x}$$

$$x^2 = 1 - x$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-1+\sqrt{5}}{2}$$

x= .618…

Golden Ratio (phi)

$$\Phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots\ldots}}}}}$$

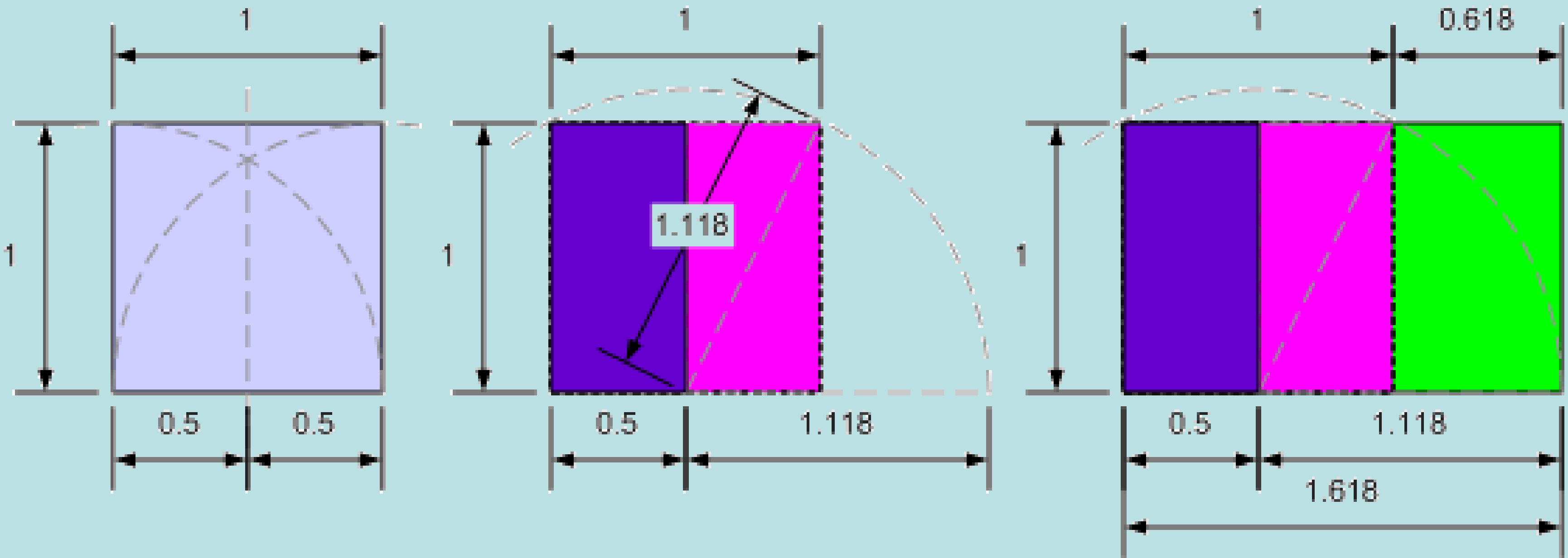
$$\text{Let } x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots\ldots}}}}}$$

$$x^2 = 1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

$$x^2 = 1 + x$$

The construction

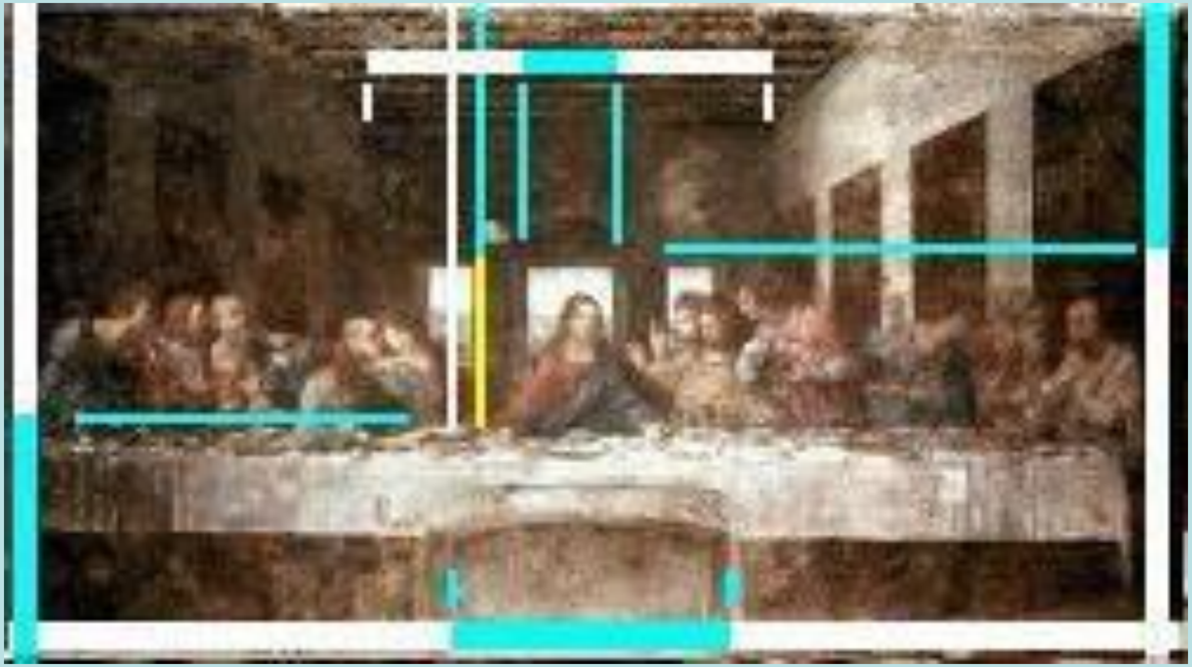
- Start with a perfect square having a length of one unit on each side.
- Bisect the square vertically.
- Draw a diagonal line from the baseline midpoint to the upper right corner of the square.
- Strike an arc from this diagonal line, using the baseline midpoint as the center of the arc.
- Extend the baseline of the square out to the right until it meets the arc.
- Draw a vertical line upward from the baseline extension.
- Extend the top line of the square out to the right until it intersects the vertical line.



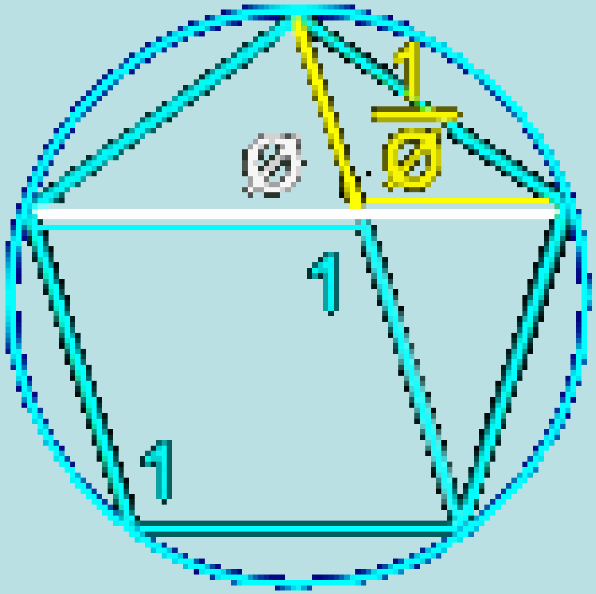
- Leonardo Fibonacci, an Italian born in 1175 AD (2) discovered the unusual properties of the numerical series that now bears his name, but it's not certain that he even realized its connection to phi and the Golden Mean.
- Da Vinci provided illustrations for a dissertation published by Luca Pacioli in 1509 entitled "De Divina Proportione" (1), perhaps the earliest reference in literature to another of its names, the "Divine Proportion."

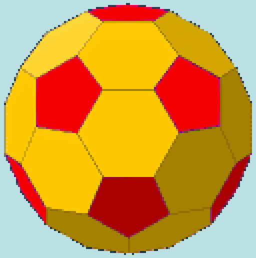
Da Vinci

The Renaissance artists used the Golden Mean extensively in their paintings and sculptures to achieve balance and beauty. Leonardo Da Vinci, for instance, used it to define all the fundamental proportions of his painting of "The Last Supper," from the dimensions of the table at which Christ and the disciples sat to the proportions of the walls and windows in the background.

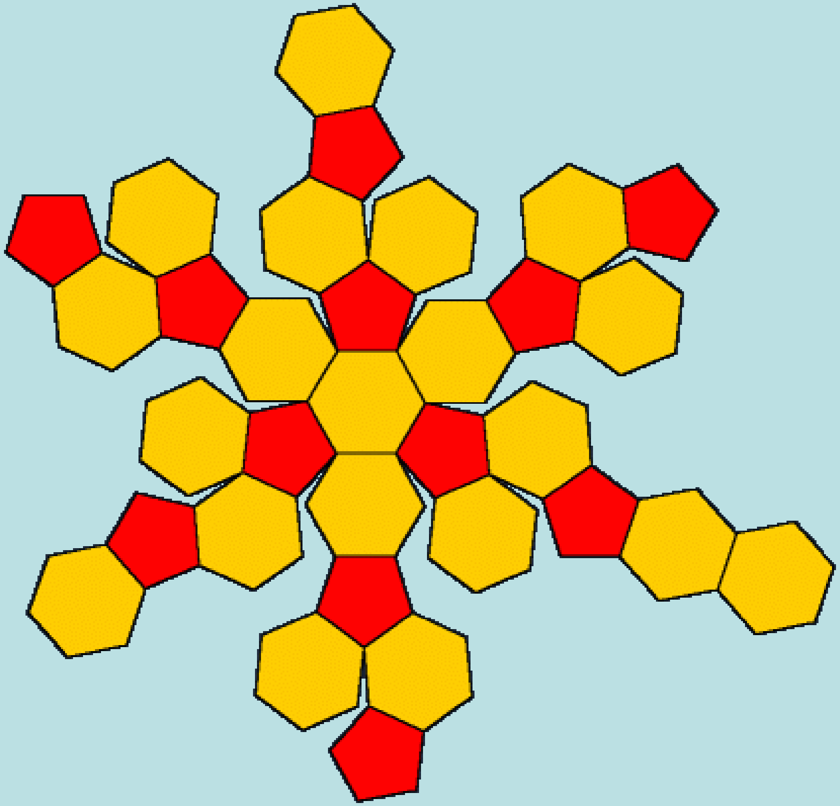


- Divide a 360° circle into 5 sections of 72° each and you get the five points of a pentagon, whose dimensions are all based on phi relationships.

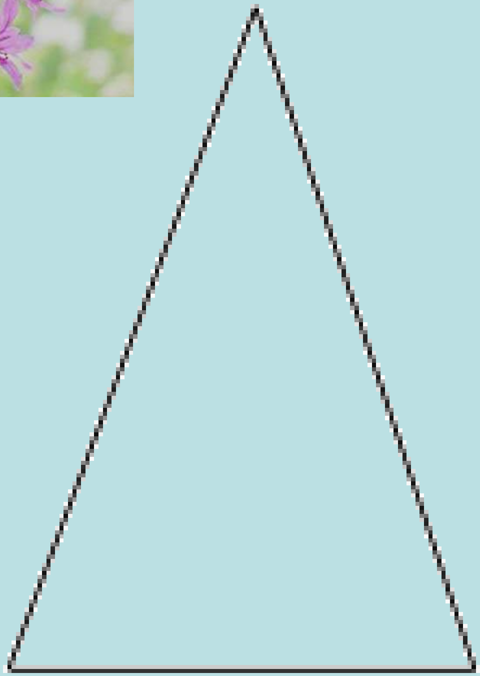




the surface consists of twelve phi-based pentagons, each one of which is connected to five of the twenty hexagons, shown unfolded below:

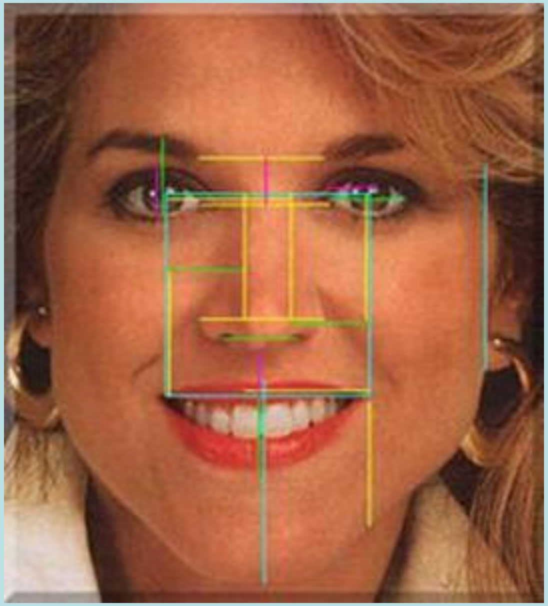


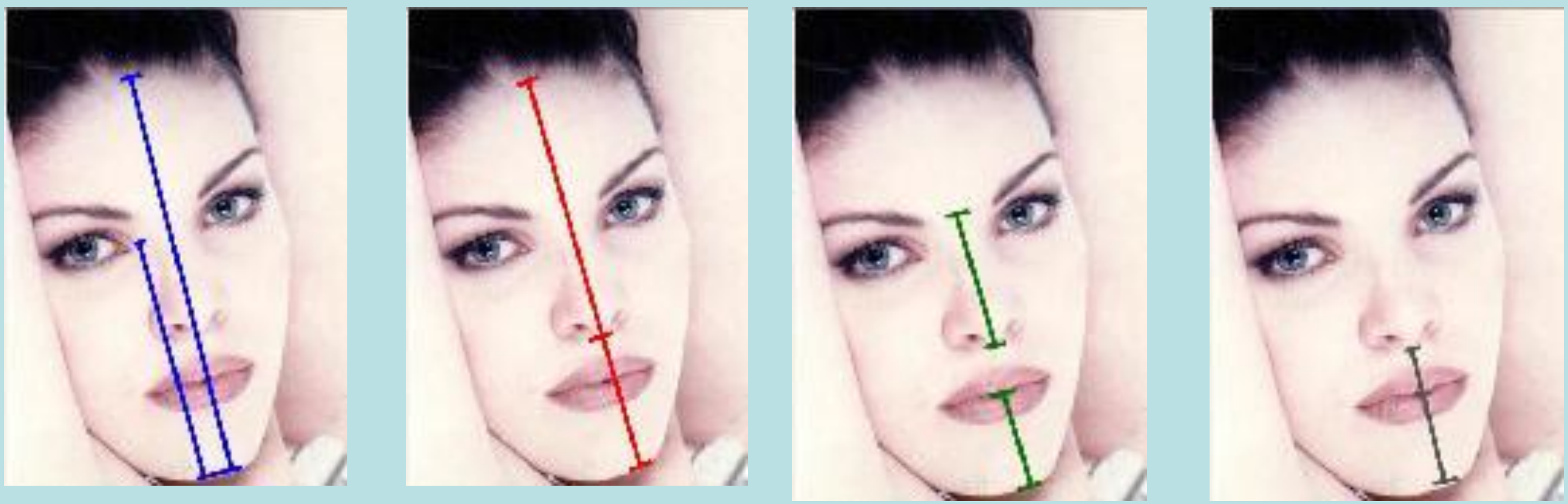
- If you draw an isosceles triangle with base angles equal to 72 degrees. Measure the length of the shorter side and the two legs, which of course have the same length since this is an isosceles triangle. What is the ratio of the lengths?



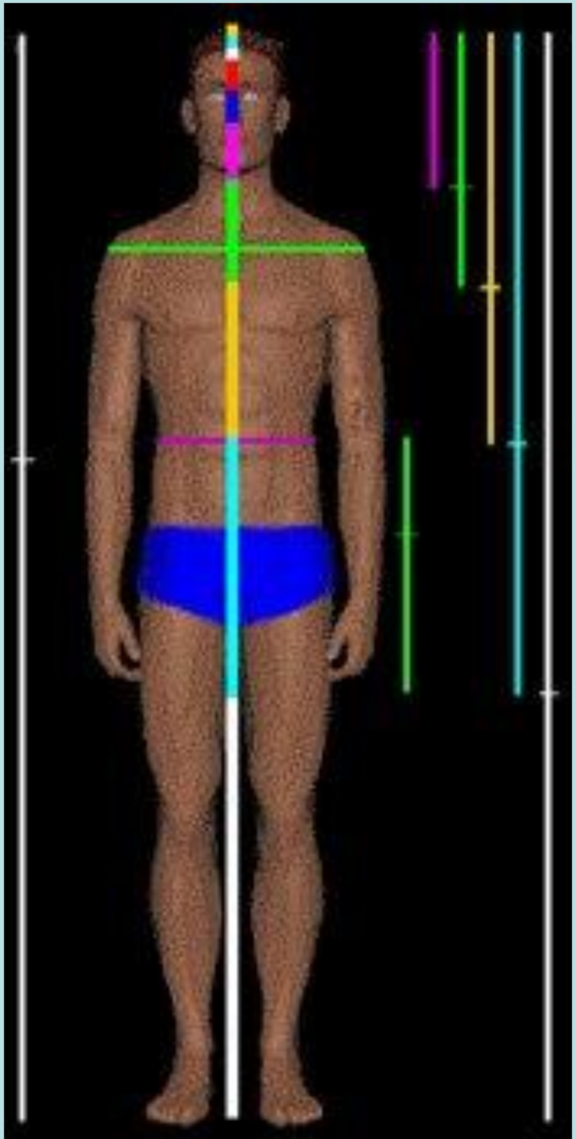
The Golden Ratio in the Human Face

- Length of face / width of face
- Distance between the lips and where the eyebrows meet / length of nose
- Length of face / distance between tip of jaw and where the eyebrows meet,
- Length of mouth / width of nose
- Width of nose / distance between nostrils
- Distance between pupils / distance between eyebrows

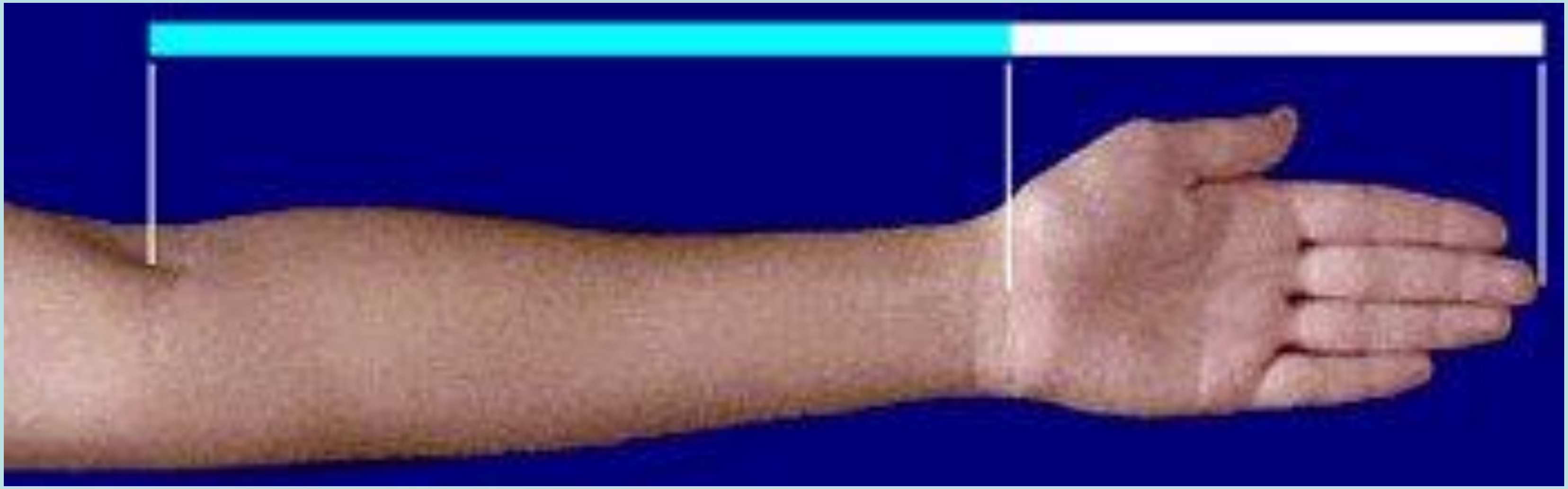




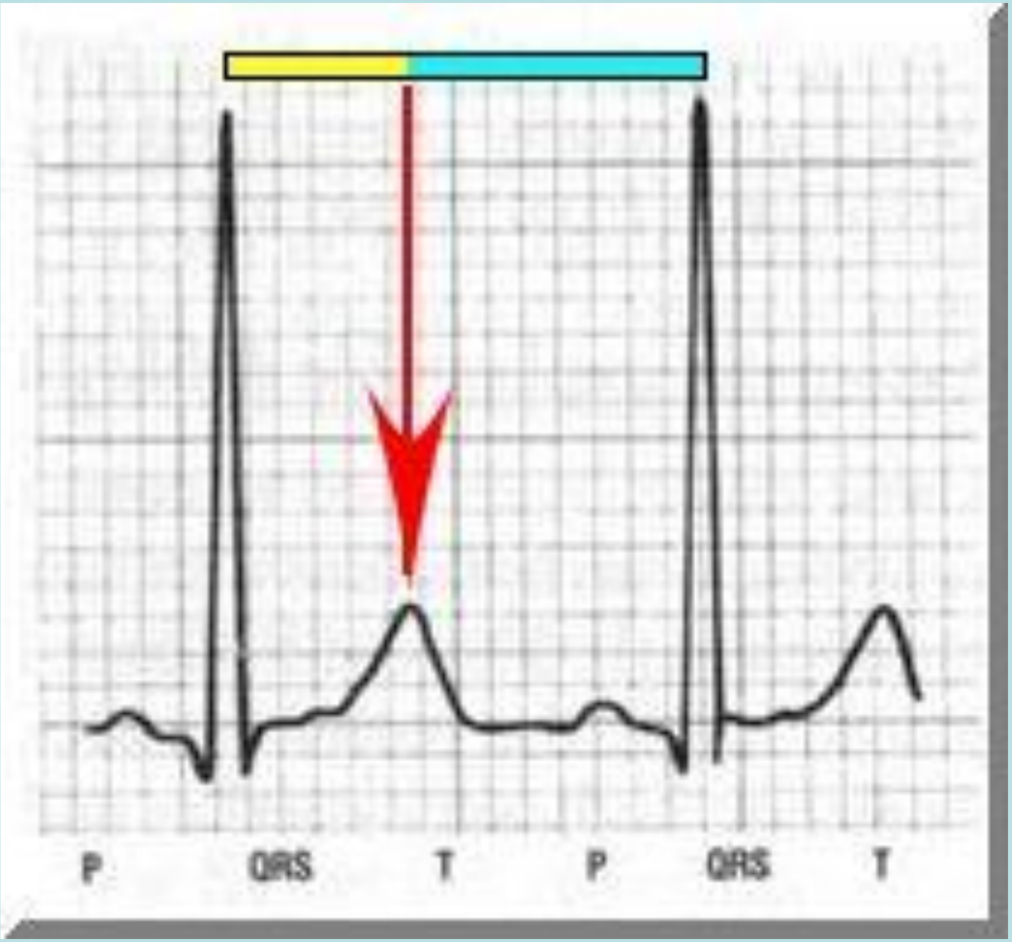
The Golden Ratio in the body

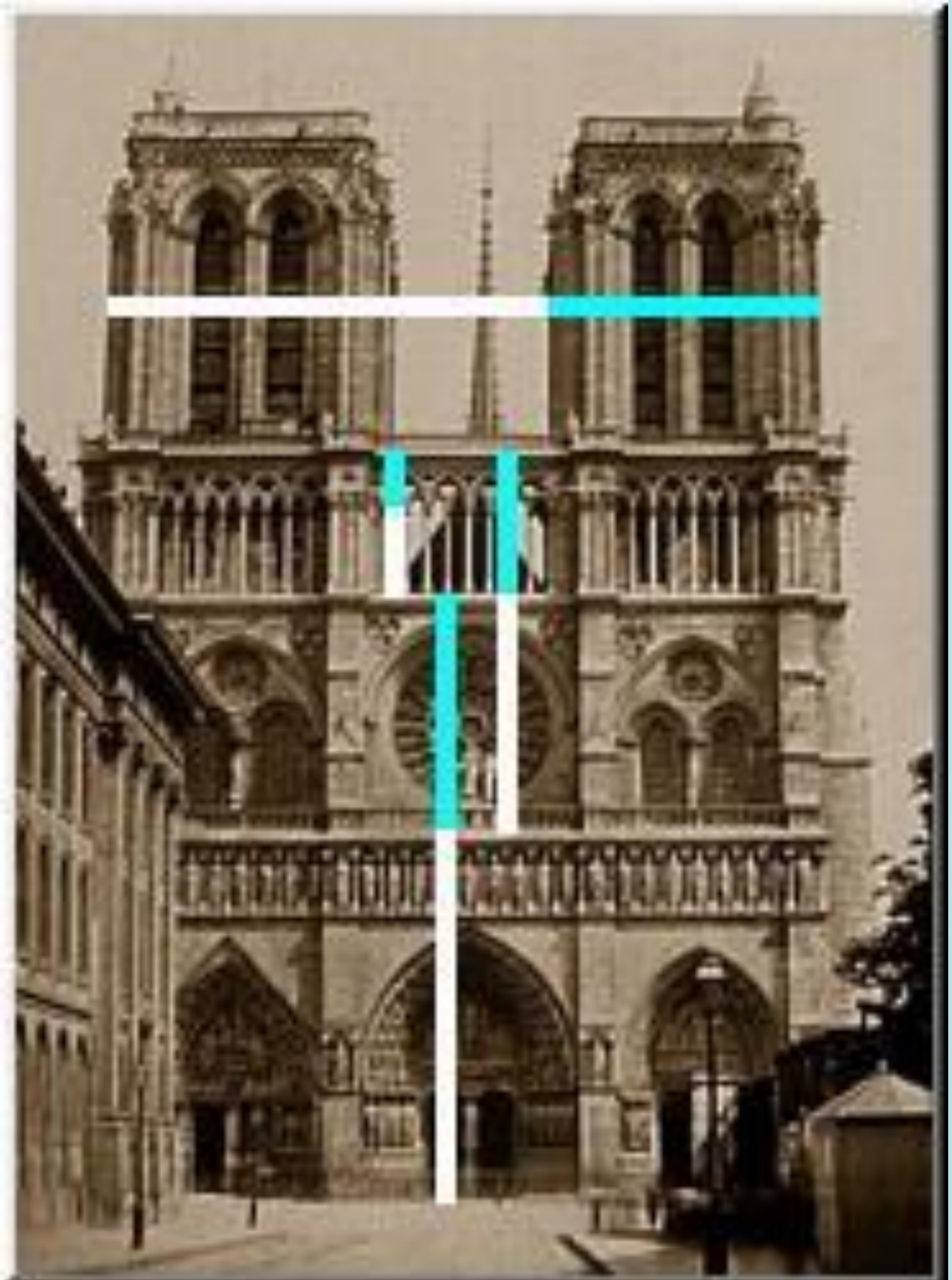


- Your hand creates a golden section in relation to your arm, as the ratio of your forearm to your hand is also 1.618, the Divine Proportion



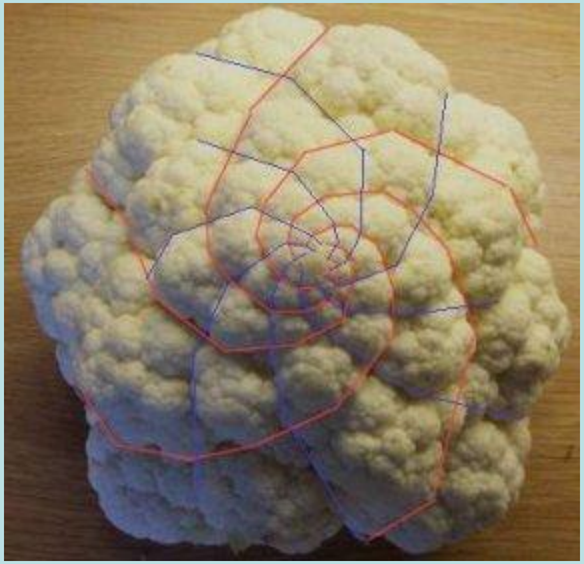
A peaceful heartbeat is said by some to beat in a Phi rhythm





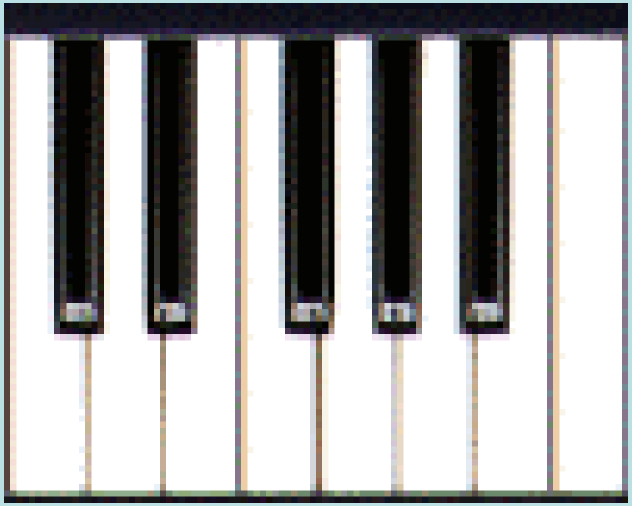

- Revelation 13:18 says the following: "This calls for wisdom. If anyone has insight, let him calculate the number of the beast, for it is a man's number. His number is 666."
- sine of 666°, you get 0.80901699, which is one half of negative phi (1.618)

Fibonacci and the Golden Ratio



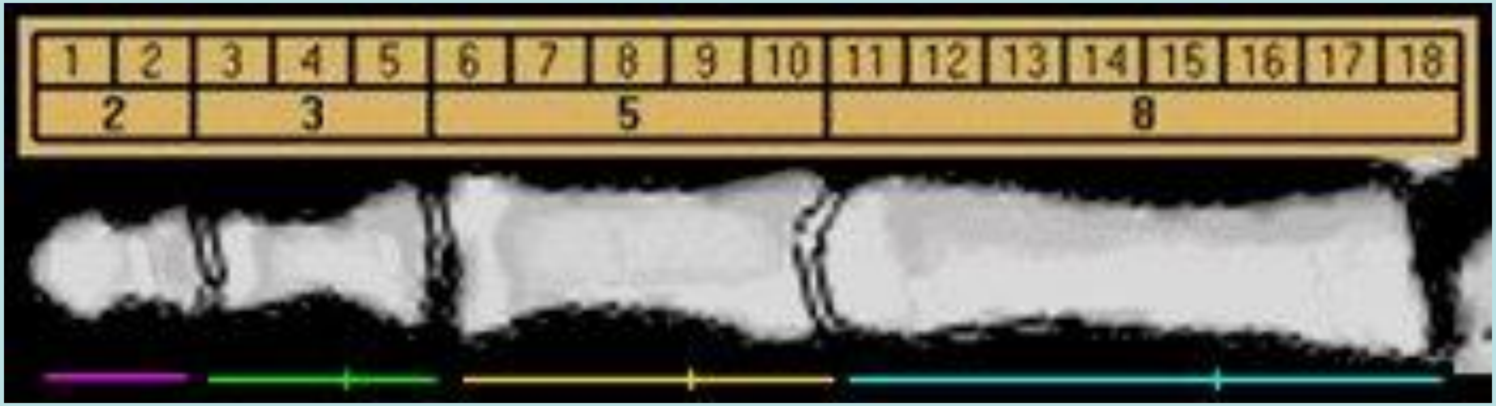
Fibonacci numbers

the piano keyboard scale of 13 keys has 8 white keys and 5 black keys, split into groups of 3 and 2.



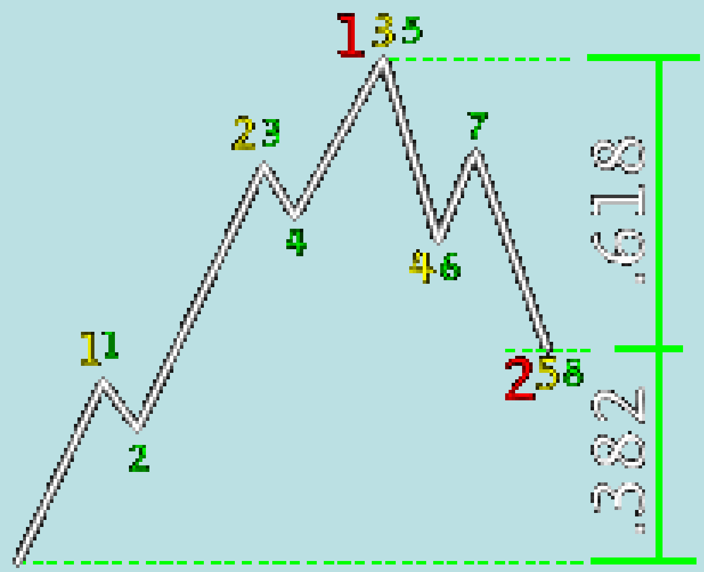
13 notes separate each octave of 8 notes in a scale, of which the 5th and 3rd notes create the basic foundation of all chords, and are based on whole tone which is 2 steps from the root tone, that is the 1st note of the scale.

- Each section of your index finger, from the tip to the base of the wrist, is larger than the preceding one by about the Fibonacci ratio of 1.618, also fitting the Fibonacci numbers 2, 3, 5 and 8.



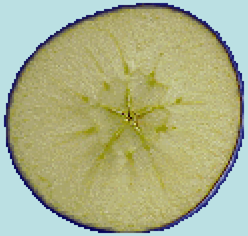
Phi and Fibonacci numbers define the price movements of stocks in Elliott Wave Theory

Major, minor and sub waves are shown in RED, YELLOW and GREEN and the total number of increases and decreases (2, 5 or 8) is a Fibonacci number. Note too that the predicted end result is based in the Fibonacci series as well as the end price is 61.8% of the high and 0.618 is equal to 1/ and 0.382 is 1/ ^{2}.



Fibonacci numbers in plant sections

Bananas have 3

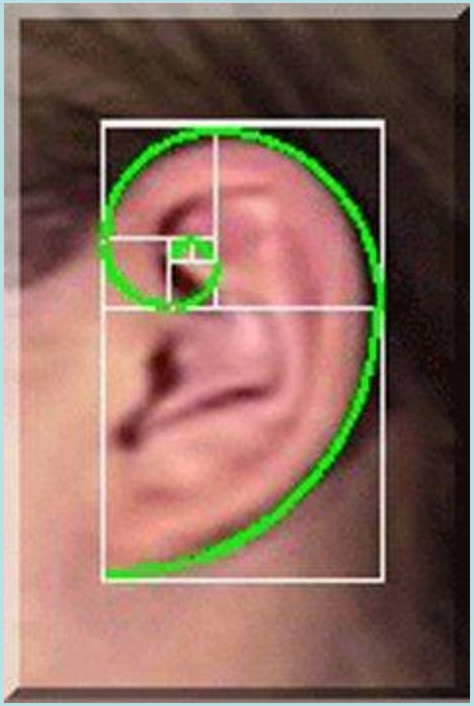


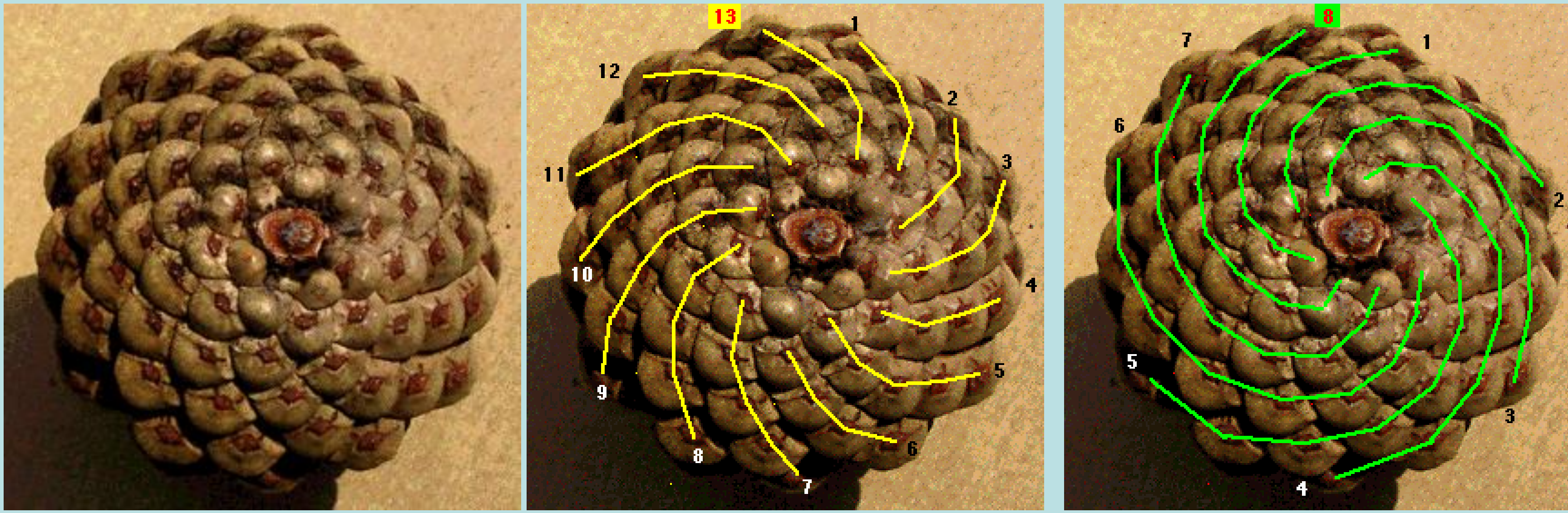
Apples have 5

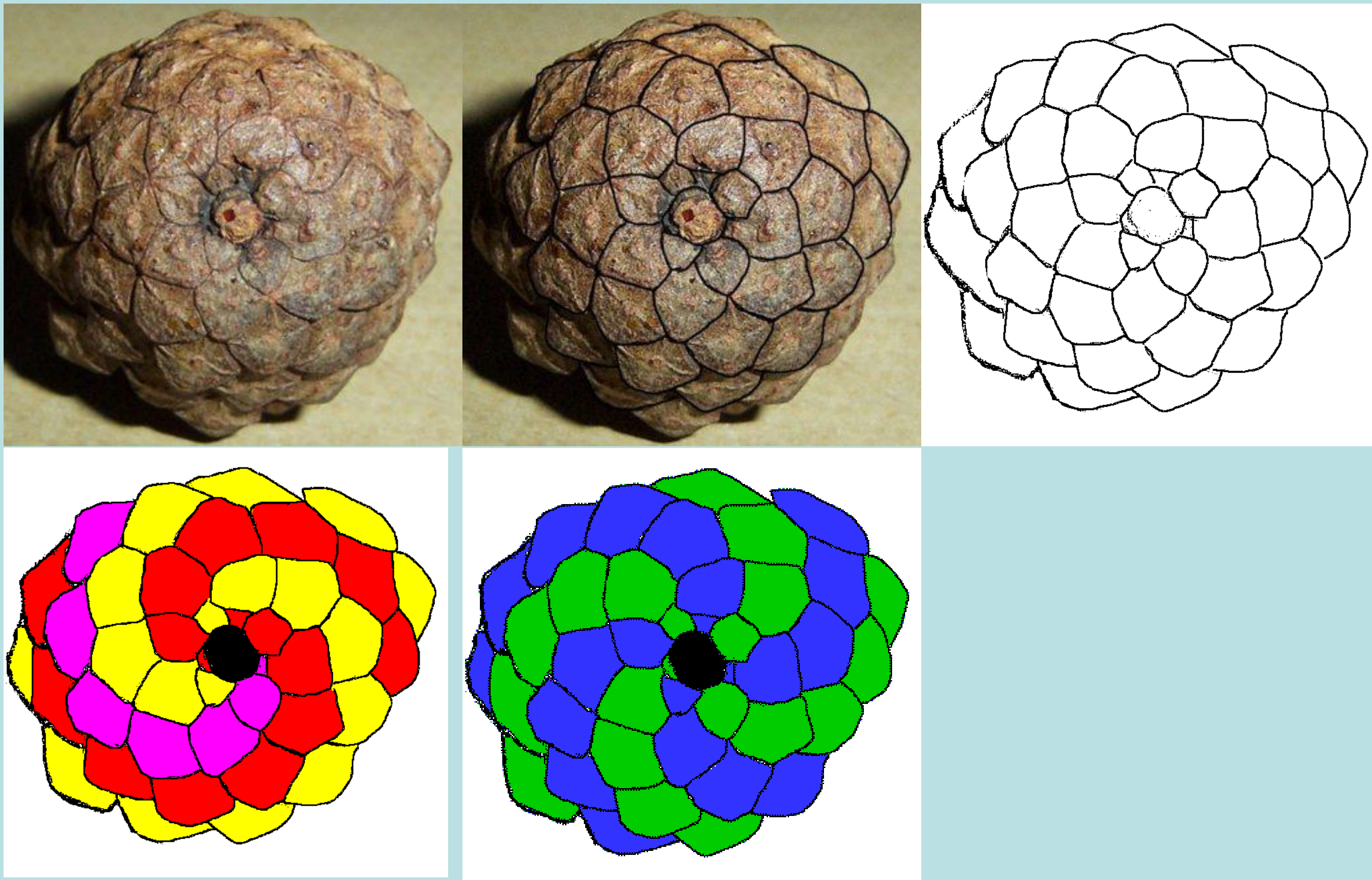
Fibonacci numbers in flower petals

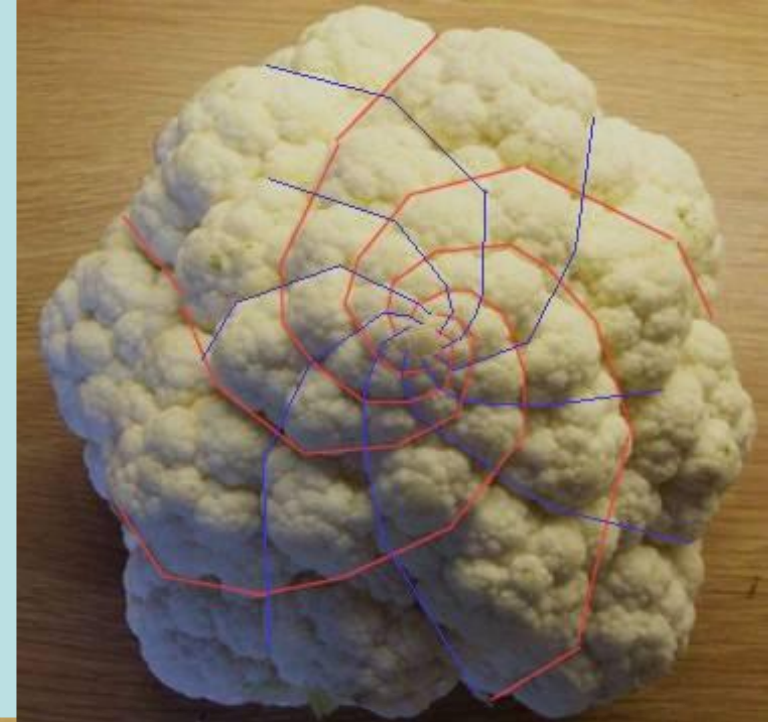
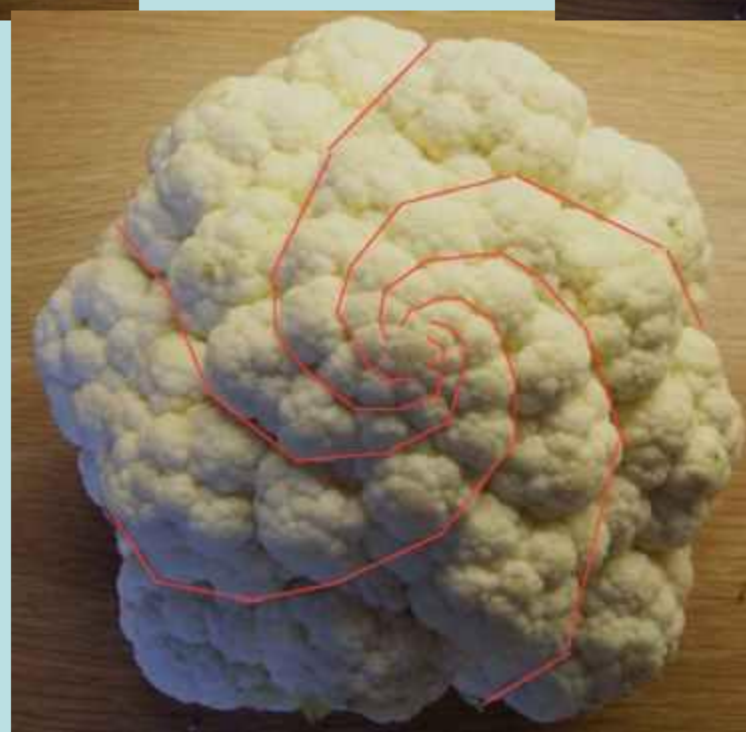
Graph paper

- The ear reflects the shape of a Fibonacci spiral.




13
1
2
3
4
5
6
7
8
9
10
11
12
8
1
2
3
4
5
6
7



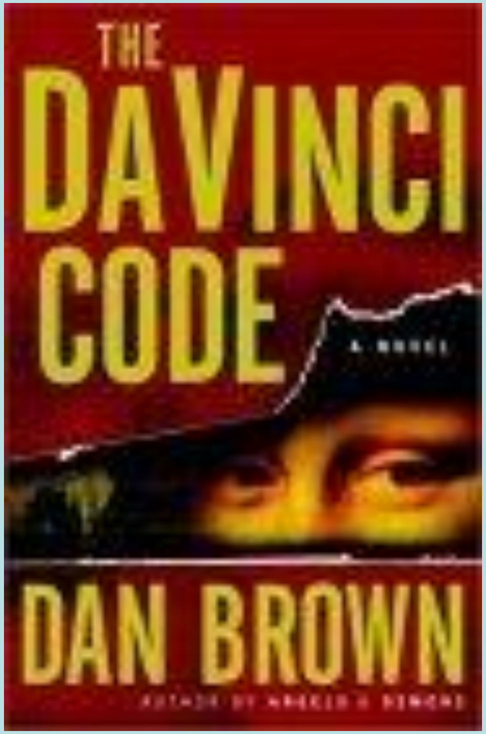




The Fibonacci series has a pattern that repeats every 24 numbers

- Numeric reduction is a technique used in analysis of numbers in which all the digits of a number are added together until only one digit remains. As an example, the numeric reduction of 256 is 4 because 2+5+6=13 and 1+3=4.
- A mathematician by the name of Jain discovered that applying numeric reduction to the Fibonacci series produces an infinite series of 24 repeating digits:

1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9

"Man is simply playing by Nature's rules, and because *art* is man's attempt to imitate beauty of the Creator's hand..."



MAY BE NOT

- **Misconceptions about the Golden ratio**
  George Markowsky's

- **How to Find the "Golden Number" without really trying**
  Roger Fischler, *Fibonacci Quarterly*, 1981

- **The Fibonacci Drawing Board Design of the Great Pyramid of Gizeh**
  Col. R S Beard in *Fibonacci Quarterly* vol 6, 1968

Your problem

A golden triangle also called the sublime triangle, is an isoceles triangle whose ratio of **leg to base** is the golden ratio.

- It is also an isoceles triangle whose ratio of **base to leg** is the golden ratio, so there are **two types**: Type I, acute, and type II, obtuse.
- Find these triangles.