Euclid

Elements

No other book except the *Bible* has been so widely translated and circulated.

In *Elements*, a set of 13 books written around 300BC, Euclid listed 5 axioms and 5 postulates from which everything else was developed. [i.e. all of his 465 propositions are proved from these 10 things … at least to Euclid's satisfaction!]

The 5 axioms have a logical base – for example

$x=y \Rightarrow x+z = y+z$ for all real numbers z – while the 5 postulates are geometric:

Euclid’s 5 Postulates

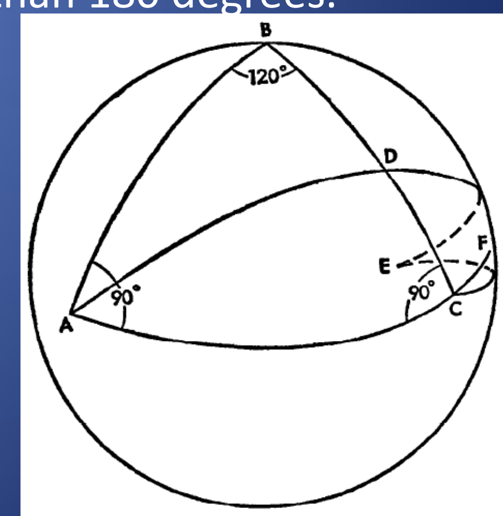
1. A line can be drawn from any point to another.
2. A finite line can be extended indefinitely.
3. A circle can be drawn centered at any point and with any chosen radius.
4. All right angles are equal to one another.

5. If a transversal falls on two lines in such a way that the interior angles on one side of the transversal are less than two right angles, then the lines will meet on that side.

The Fifth Postulate

5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

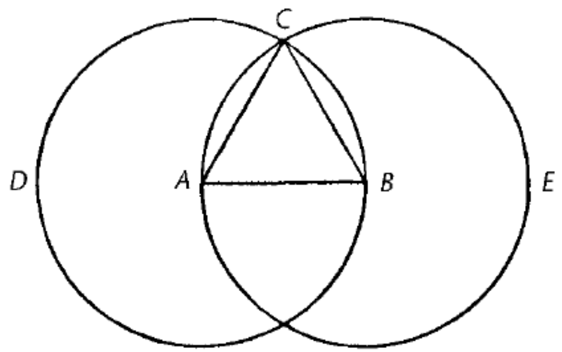
In spherical geometry, for example, this would read: ``The sum of the angles in a triangle is greater than 180 degrees.''



Proposition 1

On a given finite straight line to construct an equilateral triangle.

Let *AB* be the given finite straight line.



Thus it is required to construct an equilateral triangle on the straight line *AB*.
With centre *A* and distance *AB* let the circle *BCD* be described; **[Post. 3]**
again, with centre *B* and distance *BA* let the circle *ACE* be described; **[Post. 3]**
and from the point *C*, in which the circles cut one another, to the points *A*, *B* let the straight lines *CA*, *CB* be joined. **[Post. 1]**

Now, since the point *A* is the centre of the circle *CDB*,
AC is equal to *AB*. **[Def. 15]**

Again, since the point *B* is the centre of the circle *CAE*,
BC is equal to *BA*. **[Def. 15]**

But *CA* was also proved equal to *AB*;
therefore each of the straight lines *CA*, *CB* is equal to *AB*.

And things which are equal to the same thing are also equal to one another; **[C.N. 1]**

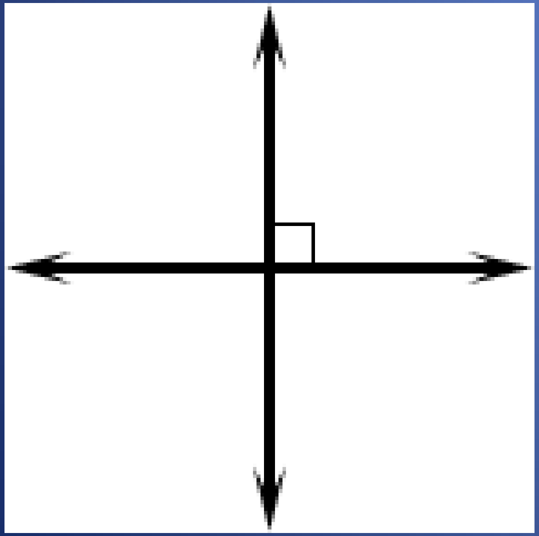
therefore CA is also equal to CB.

Therefore the three straight lines *CA*, *AB*, *BC* are equal to one another.

Therefore the triangle *ABC* is equilateral; and it has been constructed on the given finite straight line *AB*.

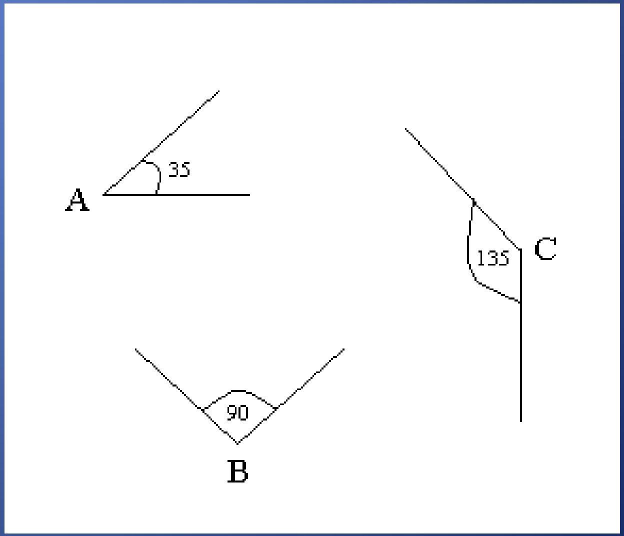
Being what it was required to do.

- **Definition 10 -** When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right,* and the straight line standing on the other is called a *perpendicular* to that on which it stands.

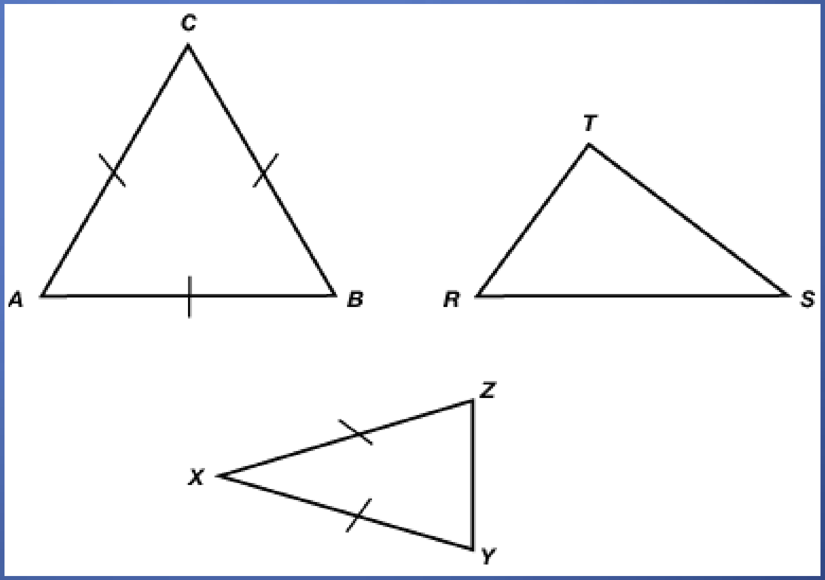


- **Definition 11**- An *obtuse angle* is an angle greater than a right angle.

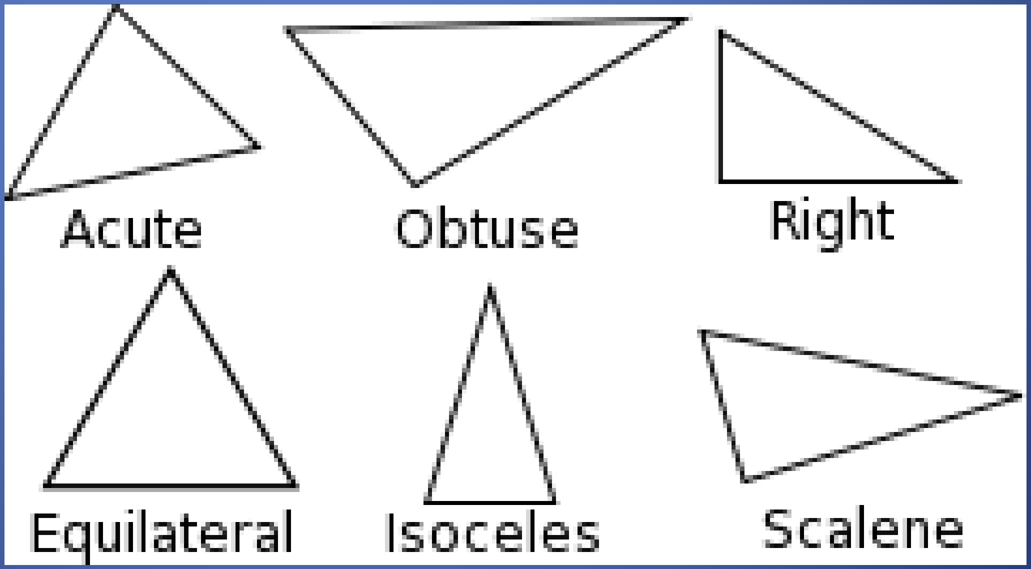
- **Definition 12**- An *acute angle* is an angle less than a right angle.



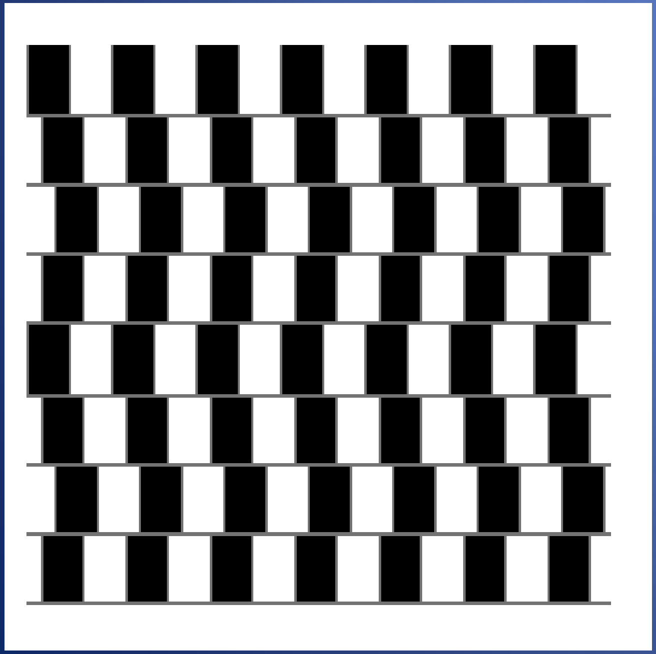
- **Definition 20** - Of trilateral figures, an *equilateral triangle* is that which has its three sides equal, an *isosceles triangle* that which has two of its sides alone equal, and a *scalene triangle* that which has its three sides unequal.



- **Definition 21** - Further, of trilateral figures, a *right-angled triangle* is that which has a right angle, an *obtuse-angled triangle* that which has an obtuse angle, and an *acute-angled triangle* that which has its three angles acute.



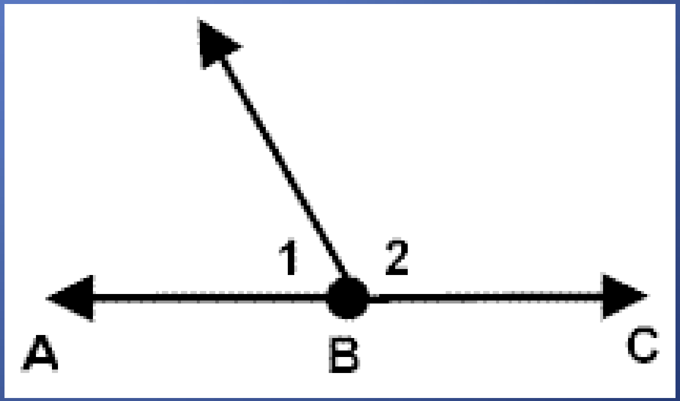
- **Definition 23** - Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.



- **Proposition 13**

  If a straight line stands on a straight line, then it makes either two right angles or angles whose sum equals two right angles.

  Those angles are considered supplementary angles.



- **Proposition 15**

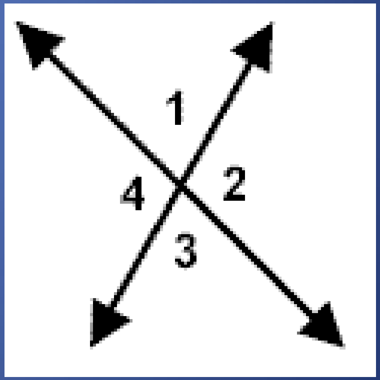
  If two straight lines cut one another, then they make the vertical angles equal to one another.

$$m\measuredangle 1 = m\measuredangle 3$$

$$m\sphericalangle 2 = m\sphericalangle 4$$

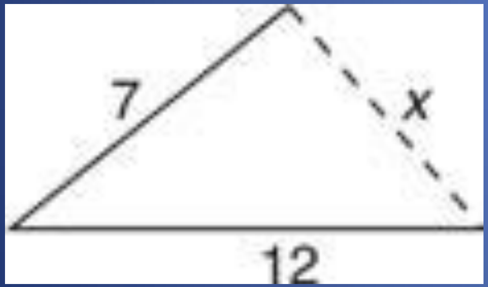
$$\measuredangle 1 \cong \measuredangle 3$$

$$\measuredangle 2 \cong \measuredangle 4$$

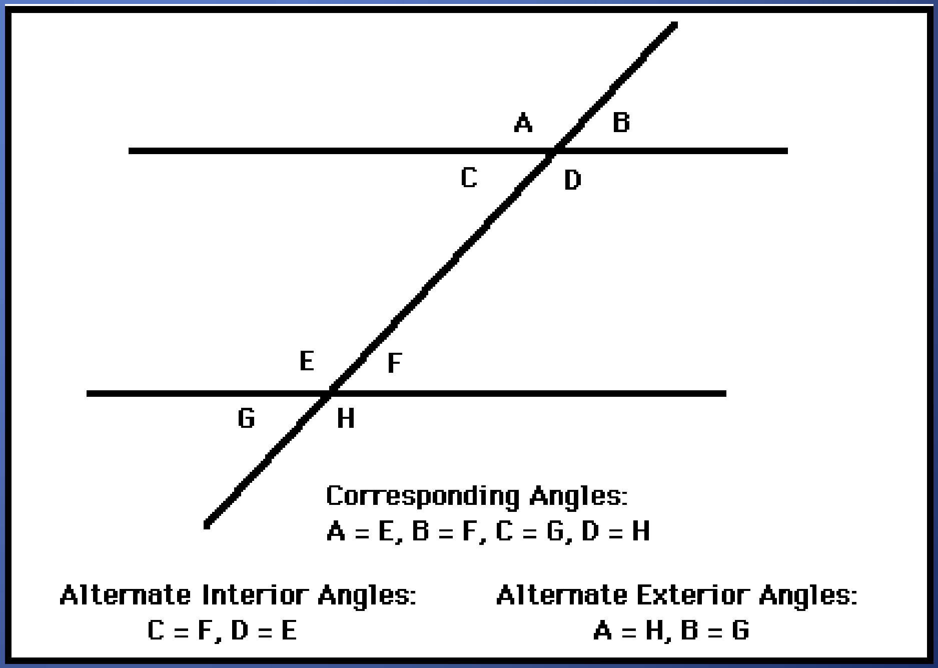


- **Proposition 20**

  In any triangle the sum of any two sides is greater than the remaining one.

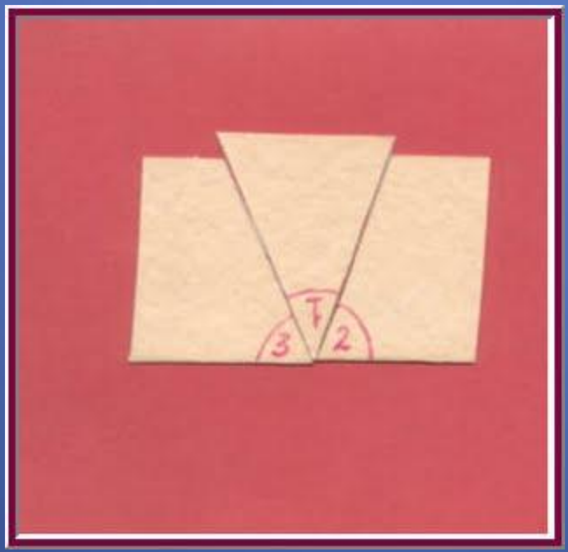


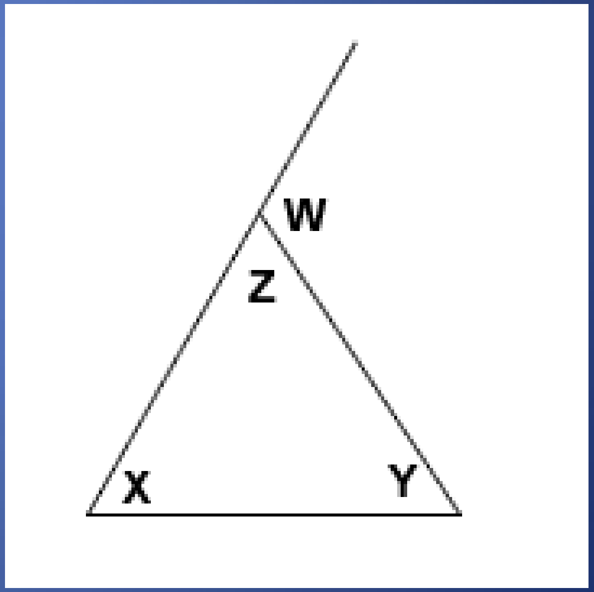
- *A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.*



- **Proposition 32**

  In any triangle, if one of the sides is produced, then the exterior angle equals the sum of the two interior and opposite angles, and the sum of the three interior angles of the triangle equals two right angles.





$$m\measuredangle w = m\measuredangle x + m\measuredangle y$$

- **Proposition 47**
- *In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.*