Ratios

Problem

A person travels 30 mph to the city, and 50 mph back. What is the average rate?

$$\frac{30+50}{2}=40$$ not true!!!!

Specific Case

$$Rate = \frac{distance}{time}$$

Assume 150 miles each way.

30 mph to the city, and 50 mph back.
5 hours to get there, 3 hours back, 8 total hours

$$\frac{300}{8} = \frac{150}{4} = \frac{75}{2} = 37.5$$

Can we write this in a general form?

$$\xrightarrow[a]{S} t_1 = \frac{distance}{time} = \frac{S}{a}$$

$$\xleftarrow[b]{S} t_2 = \frac{distance}{time} = \frac{S}{b}$$

$$\frac{2S}{\frac{S}{a} + \frac{S}{b}} = \frac{2ab}{a+b}$$

This is the harmonic mean.

- Arithmetic mean $\frac{a+b}{2}$
- Geometric mean $\sqrt{ab}$
- Harmonic mean $\frac{2ab}{a+b}$

Which is bigger the arithmetic or the harmonic?

Prove It

$$\frac{a+b}{2} \vee \frac{2ab}{a+b}$$ let us get common denominators , and compare numerators

$$(a+b)^2 \vee 4ab$$

$$a^2 + 2ab + b^2 \vee 4ab$$

$$a^2 + b^2 \vee 2ab$$

$$a^2 - 2ab + b^2 \vee 0$$

$$(a-b)^2 \vee 0$$

$(a-b)^2 \geq 0$ Therefore the arithmetic mean is always greater than the harmonic mean