Amicable and Perfect Numbers

Friendly

Two numbers are called Amicable (or friendly) if each equals to the sum of the divisors of the other

220 and 284 form the smallest pair of amicable numbers known to Pythagoras

Divisors of **220**:

1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = **284**

Divisors of **284**:

1 + 2 + 4 + 71 + 142 = **220**

Amicable History

- It was not until 1636 that the great Pierre de Fermat discovered another pair of amicable numbers (17296, 18416).
- Descartes gave the third pair of amicable numbers i.e. (9363584, 9437056) .
- In the 18th century great Euler drew up a list of 64 amiable pairs (two of which later shown to be unfriendly).
- Paganini, a 16 years Old Italian, startled the mathematical world in 1866 by announcing that the numbers 1184 and 1210 were friendly. It was the second lowest pair and had been completely overlooked until then, Even Eulers list of Amicable pairs does not contain it.
- The number were used in the Middle Ages in creating Horoscopes.

Perfect Numbers

- perfect numbers have the property that their divisors add up to the number itself.
- Smallest Perfect Number?
- Hint: It is less than 10.
- Divisors of 6 are 1, 2, and 3

$$1 + 2 + 3 = 6$$

- The next highest is ?????.
- Euclid also knew the next two perfect numbers: 496 and 8,128.

Not so perfect numbers!!

- Pythagoras observed some numbers having many proper factors, and other numbers had relatively few factors or divisors.
- Numbers that are not perfect are either abundant, or deficient.
- Abundant numbers are numbers in which the sum of the proper factors is greater than the number.
- Deficient numbers are numbers in which the sum of the proper factors is less than the number.

Determine if the first 30 integers greater than 2 are Abundant, Deficient, or Perfect.

Euclid wrote about constructing perfect numbers in this way:

If as many numbers as we please beginning from an unit be set out continuously in double proportion, until the sum of all becomes prime, and if the sum multiplied into the last make some number, the product will be perfect

WHAT?!!

- "an unit be set out continuously in double proportion": 1,2,4,8,16,32....   (1 x 2 = 2, 2 x 2 = 4, 4 x 2 = 8, etc)
- "if the sum": 1 + 2 + 4 + 8 = 15
- "until the sum of all becomes prime": simply means that we add until we get to a prime number! 1 + 2 = 3
- "the sum multiplied into the last make some number" 3 x 2 = 6
- "the product will be perfect“
- There you have it! 6 is the first perfect number!

Let’s Keep Going?

- 1 + 2 + 4 = 7 (7 is prime)
- 7 x 4 = 28: the second perfect number!
- 1 + 2 + 4 + 8 = 15 (15 not prime)
- 1 + 2 + 4 + 8 + 16 = 31 (31 is prime)
- 31 x 16 = 496 : The third perfect number!

Euclid

- On the basis of this limited evidence and some careful reasoning, Euclid proved in the final proposition on number theory in the *Elements* that if $2^n - 1$ is prime, then $[2^{(n-1)}](2^n - 1)$ is perfect.
- Try $n = 5$
- $2^5 - 1 = 31$
- $[2^{(5-1)}](2^5 - 1) = (2^4)(32-1) = (16)(31) = 496$
- Check that 496 is perfect.
- 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496

Perfect Facts

- The first four perfect numbers written in binary form also display a striking pattern: 110, 11100, 11111000, and 1111111000000
- first five perfect numbers:

  6; 28; 496; 8128; 33,550,336
- The final digits of the four perfect numbers that he knew alternate between 6 and 8. It turns out that the final digit of an even perfect number is always 6 or 8, but the alternating pattern doesn't hold.
- The issue of odd perfect numbers remains unsettled, however. No one knows whether there are any. Mathematicians have so far proved that if an odd perfect number exists, it must have at least 300 decimal digits and must have at least 29 prime factors (not necessarily distinct)